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INCREASING STUDENTS' INTEREST IN MATHEMATICS ON THE BASIS OF THE METHOD OF PEER INSTRUCTION OF THE TOPIC" TRIGONOMETRY"

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CONTENTS

Abstract

This research study aims to reveal the effectiveness of peer education techniques on 9th-class students' mathematics achievement and attitudes towards mathematics in a

trigonometry course. In addition, the effect of gender differences on mathematics accomplishment and attitude towards mathematics was investigated. The research was conducted with a total of 171 students studying at three different high schools in Almaty, Kazakhstan in the 2019-2020 school year. In the present research, one class from each of the three schools was determined as the experimental group and the other classes as the control group. There were 69 students in the treatment group and 102 students in the control group. "Peer Instruction" was used in the experimental group, and "Traditional Teaching Method" was used in the control group. In the research, quantitative and qualitative research approaches have been adopted, and pretest, post-test were used as research models. Mathematics Achievement Test, Mathematics Attitude Scale, and Peer Education Evaluation Form were used as data collection tools in the research study. The achievement test was prepared by the researcher, and the attitude scale and evaluation form were used as ready. The achievement and attitude test was applied twice, before and after the experiment. In the study, an independent sample t-test was used in the analysis of quantitative data, and an average score was used in qualitative data. The data obtained were analyzed with the SPSS 21.00 statistical program. The significance level was taken as $p < 0.05$ in the analyses. As a result of the research; there was a significant difference in the academic success and attitudes of the learners in the treatment group compared to the students in the control group. In addition, it was found that gender does not have an effect on learners' academic accomplishment and attitudes towards mathematics lessons. Participants stated that they liked mathematics lessons more thanks to peer education, and they wanted to participate more in the lesson. With peer education, their attitude towards achievement and mathematics lessons increased.

Keywords: Peer Instruction, Traditional Teaching Method, Active Learning method, Academic Achievement, Attitude.

INTRODUCTION

The main purpose of this research study is to examine the effects of peer instruction on 9th class students' academic accomplishment and attitudes towards mathematics. This research study also compares the effectiveness of the gender gap in trigonometry with the peer teaching method of traditional teaching methods on students' accomplishments and attitudes towards mathematics.

Mathematics is a system of ideas and structures improved as the process of sequential abstraction and generalizations.

In the above definition, three points are noteworthy. The first is that mathematics is a system, the latter consists of structures and relations, and the third is that these structures are formed by the process of consecutive abstractions and generalizations. So, mathematics is a system created mentally by human beings. This makes mathematics abstract. The reason why students have difficulty in mathematics is that it is more challenging to gain abstract concepts. Many subjects in mathematics can be made more attractive and concrete with peer instruction.

Rapid developments in science and technology have led to important developments in the economic, social, and cultural life of the society as the qualifications that people need to carry their educational understanding changes accordingly [1] Depending on this "Every Kazakh citizen should realize that education is the most important factor in achieving future prosperity. Education should be prioritized in the framework of young priorities. The country will be prosperous if education becomes the most essential factor in the system of values." [2]

It is seen that there is no production and structuring of new information in the past with the current and current methods. The traditional teaching method is increasingly losing its importance. Now, the education system has not been to inform the students of the education system, but to obtain and deliver information to them. The students direct their own learning by asking questions, estimating, finding, developing experiments, collecting data, and collecting the data they collected [3].

The more the training and teaching activity is addressed to the sensory organ, the more so that the learning event is permanent, the longer it is in forgetting [4]. In recent years, most of the studies on education are directed towards this. Rather than presenting the ready-made information to the student, it is based on teaching students to learn and active participation of the learners. In the center of traditional teaching, the teacher is active and the learner is the listener. Instead of assimilating and learning the information transmitted by teachers, students tend to memorize the information as it is transmitted directly [5]. E. Mazur states that in the Physics Department of Harvard University, where traditionally the course is taught by explaining and solving questions, students are unable to answer physics questions on a conceptual level, even if they solve mathematical questions [6]. Likewise, it has been emphasized that even if the students successfully learn algebraic problem

solving, the traditional teaching method does not have enough benefit for the students to understand the basic concepts of physics [7].

It is reported in the literature that students learn the concepts about the subject more accurately and permanently in courses taught with active learning strategies [8, 9, 10]. It is expressed in various researches that the lessons taught with active learning from the instructional strategies suggested by the researchers in terms of moving the constructivist theory to educational environments provide conceptual learning and provide skills that can be used in all areas of life [8, p. 39], [9, p. 12], [11, 12].

In recent years, many researchers who have seen the insufficiency of traditional teaching methods have started to develop alternative teaching methods and techniques. One of the developed alternative active learning teaching techniques and techniques is the peer teaching technique. The age of school learners is the age of participation in groups. Participation of a secondary school student in a peer or playgroup is a necessity for both the child and the socialization. This natural process is an opportunity for teachers to be used in education. Therefore, new approaches to education have been adopted. Peer instruction is one of the approaches based on group studies and learning by a discussion with peers which gives a new perspective to mathematics. This approach is an active and cooperative learning method and students are actively involved in learning processes.

It is emphasized in the literature that when the student is actively involved in learning processes, more and longer-lasting learning takes place, increases motivation, makes the attitude towards the lesson positive, and in-depth understanding takes place E. Mazur [6, p. 15], C. H. Crouch & E. Mazur [7, p. 975], T. H. Allison, J. B. Campit & R. M. Garin, F. Demirel, H. Eryilmaz, T. Gök, L. L. Lim, R. L. E. Miller, Santana-Vega & M. S. Terrell, S. S. Tokgöz [13-20].

Peer teaching has recently taken its place in the literature as an active learning method. When the literature is investigated, it can be seen that peer teaching is applied in different ways and in different disciplines. Peer teaching method; It is preferred because it is more applicable in crowded classrooms and makes conceptual learning effective compared to other active learning methods. However, this feature of the peer teaching method has also been found to be effective in environments with low student numbers [21]. In teaching environments where the peer teaching method is used, students have the opportunity to self-assess what they learn and take the lesson actively without getting bored. Conceptual questions are discussed by peers, and active participation of all participants in the class is tried to be ensured. In the lessons taught with this method; the discussions that students make to persuade their peers take the lessons out of monotony, and students are encouraged to think about concept questions. In the study presented due to these features, the effectiveness of the peer teaching method was investigated by using it in the teaching of solutions.

Peer education is a process by which, under the guidance of the instructor, one or more students of the same level are taught a concept or skill.

Peer instruction is a method developed to improve inter-student interaction and success.

The task of the teachers who apply peer instruction is to make the necessary effort to make the math lesson more popular, to turn the fear of mathematics in the students into positive, to present the mathematics to the student in a game mood, to contribute to the discovery of mathematics and to discover the mathematics by taking pleasure of them. The students develop their friendly relations with the group work they do in peer instruction and teach the subject to each other. When the peers work together more comprehensively and using a similar language, they have a positive impact on their success. In cases where peer instruction, mathematical communication increase, the self-confidence of the child increases, mathematical trust is formed in the child, and communication between children develops [22].

The target is in this research; to state the impressiveness of peer instruction in mathematics. For this purpose, 9^{th}-grade trigonometry unit subjects were selected. The influence of peer instruction on success and attitude was analyzed. In addition, it has been tried to determine whether gender has an effect on success and attitude by paying attention to gender differences while creating groups. At the end of the research, the positive and negative aspects of peer instruction were examined in detail.

Significance of the Study

The present study aimed to examine the influences of peer instruction on mathematics, which is abstract, difficult to understand and students approach with prejudice.

Mathematics is important and necessary not only for students but also for other individuals in society. Everyone should learn mathematics in order to think scientifically, keep up with technology, and solve daily life problems. Mathematics lessons are considered as difficult lesson to learn due to its abstract structure. Effective teaching methods should be preferred in order to eliminate this difficulty. With this reason, teachers have a great responsibility.

The more the training and teaching activity is addressed to the sensory organ, the more so that the learning event is permanent, the longer it is in forgetting. In recent years, most of the studies on education are directed towards this. Rather than presenting the ready-made information to the student, it is based on teaching students to learn and active participation of the learners. In the center of traditional teaching, the teacher is active, and the learner is the listener. Instead of assimilating and learning the information transmitted by teachers, students tend to memorize the information as it is transmitted directly.

In recent years, many researchers who have seen the insufficiency of traditional teaching methods have started to develop alternative teaching methods and techniques. One of the developed alternative active learning teaching techniques and techniques is the peer teaching technique. The age of secondary school learners is the age of participation in groups. Participation of a secondary school student in a peer or playgroup is a necessity for both the child and the socialization. This natural process is an opportunity for teachers to be used in education. Therefore, new approaches to education have been adopted. Peer instruction is one of the approaches based on group studies and learning by a discussion with peers, which gives a new perspective to mathematics. This approach is an active and cooperative learning method and students are actively involved in learning processes.

Many subjects in mathematics can be made more attractive and concrete with peer instruction. According to G. Akay, in the pattern of peer instruction method participants understand the topics and show their friends in an alike social community supporting them to learn as well [23]. For this purpose, it is believed that the knowledge conveyed can release more comprehensive imagery on the students' minds making it easier to comprehend and interpret. Peer instruction; rapid, enjoyable, and supportive. As a result, it has a positive impact on the achievement of learners. Participants get knowledge through action and living. Since information and ability are participants' own study, they also influence permanency in an affirmative way.

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According to research, students often prefer to talk to their friends when they have problems [24]. This is also taken into account when applying peer instruction. Students are provided with a more comfortable and fun educational environment. In addition, it has been demonstrated that peer instruction develops communication, empathy and basic help skills among students [24, p. 38], increasing their self-

confidence L. M. Brammer & G. MacDonald [25], bringing solutions to real-life problems M. D. Merrill and C. G. Gilbert [26]. According to A. J. Kola, the current generation of students needs socially interactive classes that are full of creative activities, and when learners interact, they improve their thinking skills and choose from different choices [27]. Peer instruction has been examined in many disciplines and has been observed to be efficient for growing participants' achievement and is also used to determine fields of difficulty for students in many developed countries. Peer instruction is not only useful for the student. It also contributes to the personal development of the student. C. A. Kunsch, A. K. Jitendra, and S. Sood reported positive results for the students' achievement in peer instruction with students who had difficulty in learning mathematics and who had learning difficulties. This is because peer education improves students' problem-solving skills [28].

In Kazakhstan, there is no study comparing traditional education with peer instruction and examining students' achievements and attitudes in trigonometry in mathematics lessons. In this research, peer instruction was examined in detail by using quantitative and qualitative data analysis. In the present study, the differences in peer instruction's mathematics course at the application level in student achievement and attitude are presented comparatively with the traditional teaching approach.

This research will have a guiding attribute in mathematics concerned with peer instruction method. This study would be helpful to 9th class mathematics students and teachers. It would guide the 9th-grade mathematics teachers to the learning technique they will implement to the students for an efficient learning procedure. If the treatment group is determined to be influential, they can use this in their lesson and the 9th-grade students would be made use of and they might have superior accomplishment in the topic Trigonometry they might have positive attitudes towards mathematics.

Lastly, this research would also lead as a model for forward researchers to have a similar study and improve activity research that goals to contribute to school improvement.

For this reason, schools that implement the recommended approach obtained from the outcome of this study will be able to teach students better. Managers will be advised on what should be accentuated by teachers in the school curriculum to enhance student’s accomplishments in mathematics. For the researchers, the study will support them to reveal critical areas in the educational process that many researchers were not able to investigate.

The research also wants to ascertain the effectiveness of active learning in education because the traditional method is not enough in this era to educate our students. This is because methods like the peer instruction are more engaging and beneficial for learners compared to the traditional method.

Purpose of the research

To establish the interest of students theoretically in mathematics on the basis of the method of peer teaching of the section "Trigonometry" , to develop a methodology and to conduct experiments on its effectiveness.

Object of research

The process of teaching trigonometry in algebra

Subject of the research

The use of peer instruction method in the teaching of trigonometry

The Scientific Prognosis of the Study

The effect of peer teaching on 9th grade mathematics achievement and attitudes towards mathematics;

1- The necessity of using active teaching methods in Trigonometry branch was determined.

2- It was determined that the use of peer teaching in Trigonometry branch increased the success of students and positively affected their attitudes towards mathematics.

Objectives of the research

The first objective of this study is the determination of the impact of the peer instruction method on 9th-class students' trigonometric knowledge.

The second objective of this study is the determination of the impact of the peer instruction method on 9th-class students' attitudes towards mathematics.

The third objective of this study was to determine the effect of peer instruction method on students' academic achievement in the final mathematics achievement test.

The fourth objective of the study was to investigate the gender differences in attitude and achievement in a peer instruction mathematics class and determine if there are any differences in understanding between male and female students.

Research base

This study was applied at three separated secondary schools (Suleyman Demirel College, Almaty Innovation High school and Almaty Girl Innovation High School) in Almaty and it was continued throughout the 3rd term of 2019-2020 education year.

The Research Responsibilities:

- To determine the psychological and pedagogical basis for increasing interest in mathematics through the use of peer instruction in the teaching of trigonometry in algebra;

- To suggest methods of effective use of the peer instruction in the teaching of trigonometry;

- Experimental proof of the proposed method

The main idea of the research: The introduction of a method of peer teaching in the teaching of "Trigonometry" provides an increase in the level of progress in mathematical knowledge based on increasing students' interest in mathematics.

Sources of the Research

Constitution of the Republic of Kazakhstan, Law of the Republic of Kazakhstan "On Education", State Program of Education Development in the Republic of Kazakhstan for 2011-2021, works of philosophers, psychologists, teachers, methodologists, official documents in the field of education, compulsory educational standards, plans and programs.

Research methods

Analysis of scientific-methodical, didactic and pedagogical literature on research topics, teaching methods used in mathematics lessons in schools; to examine and generalize the experience of those who study mathematics and trigonometry; formation and application of achievement test, application of attitude questionnaire, application of peer evaluation form; statistical processing and analysis of research results.

Research stages*:*

Stages I (2018-2019) - The research topic has been determined and the relevant literature has been analyzed. Studies have been done on the tests to be used in the research. A pilot study was conducted to create an achievement test in 9th-grade trigonometry. After the pilot study, analyzes were made and an achievement test was created. The attitude mathematics test and the evaluation form were determined.

Stages II (2019-2020) - The schools where the study will be conducted were interviewed and a presentation was made to the teachers who will participate in the research on peer education and how to pass the lessons. Implementation started in the third ninety of the school year. The first part of the thesis was created during the implementation.

Stages III (2017-2018) - Findings obtained during the study were analyzed. In the light of the results obtained, the second part of the thesis, the method part, was written and prepared as a source. Finally, the thesis has been prepared in accordance with the conditions.

Recommended main principles for defense:

The use of peer teaching method in mathematics lessons positively affects students' participation, success and attitudes towards mathematics.

The peer teaching model creates a positive atmosphere in mathematics lessons, so it can be used in other lessons.

Evidence and validity of the research results Comprehensive analysis of psychological and pedagogical, educational and methodological literature and textbooks on the research topic and take them as a basis for research; logical application of methods in accordance with the goals, objectives, objects, theoretical concepts, disciplines, experimental results, with theoretical, methodological and practical proof, compliance with the scientific apparatus of research; the effectiveness of the theoretical concept, the application of methods consistent with the experimental results in improving the achievement and attitudes towards mathematics and the processing of experimental results by mathematical statistics.

Research Questions

The present research has three main questions;

1. “What is the effect of peer instruction on academic achievement?”
2. “What is the effect of peer instruction on students' attitudes towards mathematics lessons?”
3. “What is the effect of gender on the 9^{th} class students’ academic achievement and attitudes towards mathematics lessons?”

In order to find responses to the research questions, the following sub-questions were formed.

Sub-questions

1. Is there a significant difference in the pretest achievement scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied?
2. Is there a significant difference in students' achievement mean scores between pretest and posttest scores in the experimental group where peer instruction is applied?
3. Is there a significant difference in students' achievement mean scores between pretest and posttest scores in the control group where traditional teaching is applied?

4. Is there a significant difference in the posttest achievement scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied?
5. Is there a significant difference in students' attitudes mean scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied in pretests scores?
6. Is there a significant difference in students' attitudes mean scores between pretest and posttest scores in the experimental group where peer instruction is applied?
7. Is there a significant difference in students' attitudes mean scores between pretest and posttest scores in the control group where traditional teaching is applied?
8. Is there a significant difference in students' attitudes mean scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied in posttests scores?
9. Is there a significant difference between the mathematics pretest achievement scores of female and male students in the experimental group where peer instruction is applied?
10. Is there a significant difference between the mathematics posttest achievement scores of female and male students in the experimental group where peer instruction is applied?
11. Is there a significant difference between the pre-attitude scores of female and male students in the experimental group where peer instruction is applied?
12. Is there a significant difference between the post-attitude scores of female and male students in the experimental group where peer instruction is applied?

Hypothesis

1. Peer instruction has a significant impact on the mathematics achievement of 9^{th} class students.
2. Peer instruction has a significant impact on the attitude towards mathematics of 9^{th} class students.
3. Gender differences have not a significant impact on 9th class students' mathematics achievement and their attitude towards mathematics.

Structure and content of the thesis:

The thesis consists of normative references, definitions, introduction, two chapters and conclusions, suggestions, reference list and appendices.

In the first section

1- Problems encountered in mathematics lessons taught with the traditional teaching method

2- Psychological and Pedagogical Problems Experienced by High School Students as a result of traditional education's failure to respond to students' achievements and attitudes towards mathematics.

3- The development of the peer instruction teaching method in the psychological and pedagogical context in mathematics teaching and methodological approaches to the problems of teaching mathematics and didactic principles of mathematics lesson using Peer Instruction were analyzed.

In the second section

1- The use of peer teaching in the mathematics lesson, its application in the lesson and the reaction of the students in the lesson where the peer teaching is applied were got.

2- The result of the Peer Teaching application in Experimental Study and the interpretation of the results together with the analysis of the results were obtained.

In conclusion section

The positive effect of using the peer teaching method in mathematics lessons on the academic success of the students was determined and the peer teaching created a nice atmosphere among the students in the lessons, as well as, the students' attitudes towards mathematics changed positively. There is a conclusion that mathematicians can apply the peer teaching method in their lessons.

Scientific novelty: To improve student academic performance and to form a positive attitude to the method of teaching mathematics to students of the same age.

- The psychological and pedagogical basis of increasing interest in mathematics is determined by using peer teaching method in the teaching of trigonometry in algebra;

- Effective use of the method of peer teaching is demanded in the teaching of trigonometry;

- The proposed method is proved by an experimental practice.

Theoretical significance of the research consists in acquiring skills in the organization and technique of using peer instruction method.

The practical significance of the research is to study the features of using the peer instruction method in teaching Trigonometry, the results of the study can be used by teachers in working with secondary students.

The reliability of the research (accuracy, reliability)

This theoretical part of this research was conducted using both local and international articles as part of the literary sources. The practical and methodological bulk of the study was done by the author in collaboration with several volunteers who took part in the experimental research.

Approbation and implementation of the main results

The findings of this research have been confirmed both in theory and in practice in several schools and universities in Kazakhstan. Several articles have also been published both locally and internationally in the light of the findings of this study.

In the conclusion of the dissertation the hypotheses of the research were confirmed and their validity proven using both theoretical arguments from literary sources and from practical experimentation in classrooms. The results were collected and carefully analyzed using credible tests and final arguments put forward giving room for the likelihood of future research.

The methods used in the work are the analysis of educational and methodical literature, comparison, generalization of pedagogical experience on the use of active learning methods and peer instruction in high schools.

The theoretical and methodological basis of the dissertation was the work of scientists, mathematicians, physicists, economists, domestic and foreign authors on the issues under study.

1 THEORETICAL ASPECTS OF INVESTIGATION

1.1 Teaching Mathematics and Problems in Teaching Mathematics

Mathematics

In the early times, mathematics emerged to respond to the basic requirements of people (agriculture, economy, military…). The seasons and calendars were prepared to determine the times when the Nile was flooded. The contribution of mathematics to our lives only is not to answer our needs. Mathematics improves the mind of man, opens his horizons. We should not only consider mathematics as advanced problems or theories. The puzzles you solve today in the newspapers, even the questions of intelligence are mathematics. They also have a system like math. So, mathematics plays an important role in our logical thinking. Although mathematics as a science has a history that is a legacy to human history, it has a long history full of events and ups and downs. There is no exact information about whether the word

"mathematics" was used in the first years of known history. Although it is not known when and where this word was formed and used, it is a fact that it is always used by people [29]. Today, every person knows and uses the word "mathematics".

Aristotle on the door of the "who does not know math cannot enter" article had hung. Even then we understand that mathematics is valued. Pythagorean's upright triangle theorem has kept its freshness to this day and the same theorem is taught today. With the addition of people coming in every century, the mathematics that has been constantly evolving has taken its form today. A definition of mathematics that is so important in our lives is still not accepted. According to M. Altun mathematics, arithmetic, algebra, geometry, such as number and size are based on quantitative properties of the sciences that examine the common name. Mathematics is a system created mentally by man [30]. This system consists of structures and relationships. Mathematical relations are the relations between structures and connect the structures [31]. Those who see mathematics as a tool are mathematics as a science that provides a continuation of human life, and those who see mathematics as purpose; they describe it as an instrument of thought and truth [32]. Mathematics is a discipline, a knowledge field, a communication tool, a way of thinking, a logical system. Besides, the information in mathematics is consecutive and built on one another [33].

As the definitions indicate, mathematics is one of the abstract concepts. This will lead the student to conduct mathematics courses to gain efficiency. In addition, those who are given daily life, the student is going to understand better than the abstract. We learned from the re-structured mathematics lessons, we learned to explore, question, and generalize. The student learns to analyze and solve the problem freely without getting stuck in the molds. It becomes a producer individual who offers free-willed students, ready for the future with talents. Mathematics is seen by people as the door opener to a good life and a good career [34]. P. Ernest stated that at the same time, mathematics is seen as a helpful element in comprehension life and the world and generating ideas about them [35].

Contrary to these ideas, S. Poisson said to stress the importance of mathematics, “There are two things worth living in life; discovering and teaching mathematics." [36]. Views parallel to this idea have become dominant today. For this reason, the opinion that mathematics discipline should be acquired by every student still remains valid. Even in the US, studies based on the "Mathematics for All" principle continue as intensely as possible. The reason for this is that the dizzying technological developments need mathematical knowledge to continue and use them. Therefore, it is still valid to provide at least basic mathematical knowledge to our children. Therefore, changes in mathematics curricula can be seen as one of the steps taken in this aspect.

Trigonometry

Trigonometry is one of the branches of mathematics. The teaching of trigonometry is a crucial part of the development of mathematical language and

mathematical thinking. We can say that those who have learned trigonometry subjects will have the power to make good comments and gain the ability to synthesize their knowledge and implement it. One of the most challenging factors of our country's education system is the teaching of mathematics. The idea of "mathematics is difficult", which the people of our country continue to transfer from generation to generation, and the attitudes and behaviors of the teachers of Mathematics at the point of teaching reinforce the cold approach to trigonometry. New developments in the teaching methods of mathematics subjects should be monitored very well by teachers. Students need to be raised as productive generations who seek answers to the questions of why and why, who argue, who go to the conclusion with their thoughts, and thus increase self-confidence. Teachers must believe first that there would be no teaching of trigonometry by memorizing the formula. It is a well - known fact that the purpose of mathematics is not to train live calculators, but to train productive people who think, can debate and transfer the acquired knowledge to life, can generalize, try to solve problems with mathematical thinking, are far from memorizing. It is a dream to think that individuals without mathematical thinking can help positive developments in a world that seeks peace and tranquility. It is not possible to distinguish the difficulties encountered in teaching trigonometry from the general teaching of mathematics. In today's conditions, the situation of curriculum programs, overcrowding of classes due to the group work cannot be done enough, student-centered education request cannot find an application environment, lack of use of techniques of the information age, lack of teacher training programs, the structure of university entrance exams, trigonometry teaching and learning is difficult to fit the purpose. These are the challenges of general education teaching in all branches. In particular, the efficiency of the traditional method of expression and question and answer has been demonstrated by the research of educational scientists. In the researches, it was observed that the learning rates of a subject (15% by listening, 35% by listening and seeing, 85% by doing) and it was concluded that the Chinese proverb "I forget what I hear, I remember what I see, I learn what I do" may be a basis in learning [37]. In reality, there is no way of life without Mathematics. Because mathematics is a rational thought system, and it exists everywhere human beings exist. Mathematics is the mother of Science [38]. The teaching of mathematics provides individuals with the ability to generalize by giving them the habit of rational, original, clear, and intuitive thinking. It is one of the purposes of mathematics education to train individuals who have developed aesthetic aspects and who are skilled in exhibiting behaviors that contribute to positive developments. In this context, trigonometry information has become used continuously in the development of the environment in which we live. "Teaching, is the process of educating, guiding and realization of the agreed behaviors" [39]. When it comes to teaching mathematics, it always comes to mind. Any issue cannot be fully separated from other issues. This is explained by Y. Ersoy and et. all., "mathematics is a stacked science" [40]. H. Sulak expressed the same situation, "Mathematics is a network of interconnected concepts and thoughts [41]. In mathematics teaching, each subject has a close relationship with the subjects that

precede it. The new subject cannot be learned if the information in the pre-requisite position is not learned. You can't expect it to be permanent in your memorized knowledge. T. Terzioğlu explained the same expression "mathematics is cumulative " [42].

In the Basic Law of National Education, the purpose and the processing of mathematics courses are clear, but it deviates from these objectives and the way they are processed in high school applications. The student's attitude towards trigonometry subjects, whose main purpose is to enter the University, forces the teacher to explain the lesson in a certain pattern in mathematics class. The aim of trigonometry teaching contradicts the aim of the students who strive to go from shortcut to conclusion by memorizing the formula. When the style of course processing in private classrooms and the attitude of the parents and the desire of the teacher to call himself a good teacher combine, a mass of students who memorize the formula without even hearing the name of the subject and try to learn by force is formed. To achieve success, the student tries to learn trigonometry by memorizing the formulas in the journals and textbooks to solve the problems that are appropriate to them. Reason and effect relations do not need research, and even sees the process of proof as a waste of time. H. Alkan, M. Sezer, Z. South, AZ. Ozcelik, H. Koroglu stated this situation as a forced education system [38, p. 52]. Also, A. Baki et. all., named as transactional opinion [43].

Many of its mathematical concepts are abstract concepts that require a high level of cognitive activity. Everyone must admit that concepts that are more concrete and less abstract are easier to learn [44].

The teaching and learning of trigonometry subjects require patience. After an important accumulation of knowledge is formed, speaking in the language of mathematics develops spontaneously. It is necessary to repeat the information with appropriate techniques and methods. "Mathematical thinking, making generalizations by original thinking and applying what they have learned to live" in the subjects of the teacher and the student should strive tirelessly. Learning the language of mathematics is similar to a baby learning to speak. He listens constantly to what the mother, father, and his immediate surroundings say, pays attention to what is said, and stores what is said in his brain like a tape recorder. The mother, father, and other people who care for the baby do not give up their preoccupation with the baby because they do not understand what they say or because the baby cannot respond. There is a time when the baby suddenly starts speaking with the language of the people around him. That's how mathematics is taught. Teachers should always renew themselves by following new developments so that students can speak the language of mathematics well. In the teaching of trigonometry, which is a part of the teaching of mathematics, the importance of the language of mathematics is great, because in this section, there are many encountered problems that have more variables. In recent years, research on the solution of these problems has gained intensity. Adaptation and association of concepts should be taught, students should develop the ability to predict and interpret. When trigonometry subjects are not taught with appropriate

tools and equipment, permanent learning can occur. It is necessary to train people who think, produce, and generalize instead of people who memorize and memorize as much as they hear. Students should develop their mathematical and expressive skills by giving theoretical knowledge and practice together and in harmony, without drowning the student in the confusion of concepts. It is necessary to discourage students from the mindset of memorizing the formula and to direct them to fall, and to interpret it, thus leading them to speak in the language of mathematics, which is the easiest way to learn trigonometry. It is also clear that this full learning environment can be created with teachers who have learned to listen to their students.

Teaching Mathematics

All civilizations have given great importance to mathematics. In almost every country's education system, mathematics teaching is as important as the main language teaching. S. Poisson emphasizes the importance of mathematics "There are two things that are worth living in life; teaching mathematics and teaching mathematics" [36, p. 197]. Today, the idea of gaining mathematical discipline has become dominant in every student. Even in America, Mathematics for All, studies are carried out. The reason for this is that mathematics is considered to be a means of science and technological developments beyond facilitating everyday life.

The mathematics course aims to educate people with abstraction power. For example; when we encounter a problem in mathematics, we try to understand the problem first. We then examine the relationship between what is given and what is desired. If a relationship cannot be found, we get help from some helper problems. Finally, we need to have a way or a plan for the solution. We implement the plan and examine the solution we have achieved. These simple steps in the solution of mathematical problems include concepts such as research, intuition, creativity, and discovery, which are the basic elements of abstract thinking. In this way, mathematics develops abstract thinking in humans.

The general aim of teaching mathematics is according to M. Altun: "To give the person the arithmetical skills and knowledge essential for daily life, to teach him / her problem solving and to give a way of thinking which deals with events in problem-solving approach." It is seen that a student equipped with mathematics skills can express his / her thoughts clearly, think independently, and systematically organize data [30, p. 17].

Mathematics, which is a system of thought and a global language, is a very important aspect for the individual, society, science, and technology in today's developing world. Mathematics is a field that is essential for the development of behaviors such as resolving, communicating, generalizing, creative and independent thinking in daily life, job, and profession.

Conceptual and Operational Knowledge in Mathematics

Students use conceptual and operational information to solve mathematical questions. Also suitable for the structure of mathematics teaching students;

- Understanding the concepts related to mathematics (Conceptual knowledge of mathematics)
- Understand mathematical processes (Procedural knowledge of mathematics)
- Be able to establish the relationship between concepts and processes (Connections between conceptual and procedural knowledge) [31, p. 60].

Mathematics was formed structure and concepts. As mathematical concepts are abstract concepts, it becomes difficult to learn by students. This is one of the reasons why mathematics is difficult for students. For this reason, teaching the concepts correctly in mathematics is of great importance.

When the secondary school mathematics program is examined, it is seen that there is a chain structure among the subjects. Learning a new topic is linked to information from previous issues. For this reason, information learned from previous subjects is a prerequisite for new subjects [30, p. 19]. When new knowledge can be appropriately associated with old knowledge, then the meaning of the concept in question becomes apparent [45]. The knowledge of the operations is defined as the symbols used in mathematics, rules, and knowledge of the procedures that are applied when doing mathematics [44, p. 21].

According to J. Van de Walle, K. S. Karp and J. M. Bay-Williams, teaching appropriate to the structure of Mathematics must be for three purposes:

1. To understand the concepts of mathematics (conceptual knowledge),

2. To understand procedural knowledge, (procedural knowledge)

3. To help them establish connections between concepts and processes (connections) [46].

The understanding of mathematical knowledge is to link operational and conceptual information with each other. Most learners think that mathematics is a process that needs to be memorized. They are not aware of the fact that there are concepts based on the processes they use and what mathematics means. Conceptual information and operational information cannot be separated from each other. For example; the student, who knows that the area of the parallelogram is the product of the base length and the height, only used operational information. But by resembling the parallelogram to the rectangle it had learned earlier, the student who created the field formula provided both meaningful and permanent learning. In education, the task of teachers is to educate students who think, question, and associate what they learn with what they do, not to direct the students to heart. For this reason, traditional approaches to education have been replaced by new approaches.

The concept of method has been defined in different ways to date. A method is an organized way that is consciously chosen and followed to achieve goals such as

solving a problem, concluding an experiment, learning or teaching a subject. The chosen method should help us reach the goal in the most accurate, easiest and shortest time possible.

Teaching methods are classified in different ways. In order to determine the differences in terms of development, the methods are examined in two groups as traditional and contemporary.

In traditional teaching methods, all activities are shaped according to the view that the teacher is at the center. In these methods, the teacher is the active receiver and the learner is the passive receiver. All roles are gathered in the teacher. Group teaching is in question. Verbal interaction in the classroom carries great weight. In modern education systems, the student is active. Teaching is done visually rather than verbally.

There is more learning by doing and experiencing. The duties of the teacher are also responsible for facilitating the learning of the student, guiding the student and constantly motivating the student. Instead of dealing with what is presented to the student, what the student does has become more important. With modern teaching methods, it is tried to ensure that the student learns by himself, adjusts his time according to himself, and interacts directly with the learning source. As a result of the rapid progress in computer and communication technologies, it is important for the student to reach and configure secondary information on their own. In such approaches, the teacher is in the position of a guide that guides and guides the student.

The choice of method is of great importance in gaining desired behaviors in students in the learning and teaching process. Effective communication with students can be achieved by choosing a method suitable for the content. In order to ensure success in choosing the method, the cognitive and affective input behaviors, mental development level, and motivation level of the student should also be taken into consideration. In method selection, method selection should be made by taking into account which method is successful in which level and subject by making use of the research results.

In a world where science and technology are changing rapidly, the importance of mathematics is increasing and it is cared for by all people. Many learners put their focus on mathematics because they believe it will help them to be productive citizens, to solve several personal and professional issues, to understand social events, and to have a worthwhile job [47]. However, mathematics, which is sometimes defined as "difficult", "boring" and "not fun" for students, is evaluated as a "difficult to teach" and "low student interest" course for teachers [48]. The basis of these perceptions is that the teaching methods and practices used in the classroom are inadequate or completely wrong. The more the students experience about mathematics, the less their fears and anxieties decrease, and their positive attitudes increase. For this reason, we need to explain very well to our students that we should not be afraid of

mathematics [49]. However, it is almost impossible to ensure that mathematics is a popular course and to explain the importance and necessity of mathematics to students with the traditional method of expression.

In general, teaching models are divided into two main classes. These are called teacher-centered or authoritative and student-centered. Sometimes, in practice, the applications of teacher-centered models can also be called traditional teaching methods.

Traditional Teaching: It is a form of practice in which methods such as lecture, question-answer and discussion are used under the leadership of the teacher. However, the principles on which this practice is based and which learning theory its practitioners consciously consider are not fully stated.

• The traditional technique involves the teacher taking control of the class in terms of lesson flow, mode and pace of learning and even the method of testing and assessing students

• In a teacher-centered learning environment, the students are assumed to have a blank slate therefore, there is need to transfer the teacher's knowledge to them. The assumption is therefore that the information being passed is absorbed by the participants as it is being conveyed.

• In this technique of education, there is no specific way to determine the extent of learning; what and how much they have learned is not considered. To put in another way, the role of the students here is to wait for the teacher to convey the essential knowledge in a way that is convenient for them.

It is striking that teacher-centered, traditional teaching is widely practiced in our country. Traditional understanding accustoms students to readiness, directs them to memorization, reduce their sense of curiosity; It leads to the growth of individuals who do not question and therefore do not produce. However, today's conditions necessitate the training of people who reach and use information and question the information they have acquired. Knowing is not enough, it is necessary to apply; Asking is not enough, action is required. Because knowledge comes to life and develops through action.

These statements reflect general perspectives on practices called traditional teaching. What is the prevalence level of this form of practice, which is generally compared with its alternatives in educational research and which is expected to be changed? Are all or most teachers the protagonist of such a teaching activity? Actually, this is the first question to be asked. The second question to ask is to what extent are the alternatives viable?

Claimed Weaknesses of Traditional Teaching

In our age, when the causes of the problems encountered in education are investigated, it is stated that these are mostly caused by traditional teaching.

Today, where the necessity of raising students as individuals who think logically and creatively, question, research, solve problems and take responsibility for their own learning is emphasized by all education researchers, the weaknesses of traditional teaching are listed as follows:

• A teaching approach that focuses on transferring knowledge

• The instructor is seen as the only authority in the classroom.

• Teaching methods are dominant, in which students accept the ready-made information without questioning them, and where interpretation, personal views and creative thoughts are not included.

• Individual differences between students and their learning needs are not taken into account.

• There is excessive dependence on textbooks.

• Students are not encouraged to research, they do not make an effort to reach information.

• During the evaluation phase, the students send back the information conveyed to them without comment.

• Interaction and information exchange in the classroom is very limited. Therefore, it also slows down the development of students' social aspects.

• The student is directed to memorization, not to study.

• The student does not question the information he has acquired, and does not investigate the reason.

• It is very difficult to motivate the student who attends the lesson as a passive listener, to attract his interest in the lesson and to keep his attention for a long time.

Despite all these weaknesses, the question to be considered arises by itself:

“Why do teachers prefer a teaching application with the above-mentioned features?”

1. Classes are crowded.

2. Curriculum structure of the courses.

3. Teachers' inability to adequately comprehend teaching methods in the institutions where they are trained.

4. Easy and effortless.

5. To being more economical.

6. Insufficient follow-up of new teaching approaches by teachers.

In addition to these, a lot of information can be transferred in a short time; The fact that it is an effective method in introducing students to a new subject, repeating the subjects and summarizing the subjects can be counted among the reasons why the traditional teaching method is preferred by the teachers.

It is very important to train people who produce information instead of memorizing information. For this, first of all, raising individuals who have learned learning itself correctly and teachers who believe that the teacher who knows how to teach can also be learned should be the first step to find a solution to the problem. In this context, the role of instructional design gains importance.

In traditional lessons, it can be seen that students generally stay in the position of passive observers, sometimes they only copy what the teacher writes on the blackboard into their notebooks, and sometimes they remain in the position of listeners and lose their motivation after a while and leave the lesson environment completely [50], [51]. Students who are not encouraged to actively participate in the lesson do not make an effort to understand abstract concepts in depth and prefer to memorize information and formulas that will only be useful to them to solve the questions that may arise in the exam. Thus, students focus on the solution of questions containing only certain information and formulas instead of in-depth and conceptual learning expected from them.

It is observed that students who attend their classes with traditional methods are generally unable to answer conceptual questions about the basic concepts that form the basis of these formulas, as they are competent in answering questions based on certain formulas in courses such as numerically weighted physics, chemistry and mathematics [52], [53]. At the point of not being able to answer the conceptual questions at the desired level, it is thought that the traditional teaching method does not sufficiently direct the students to think, analyze and synthesize, and tries to turn them into a question-solving machine only [54].

In order for students to learn more accurately and conceptually, appropriate teaching methods and techniques should be used in mathematics education. It is reported in the literature that students who are left only in a passive repetitive position in the traditional teaching method, achieve more effective and in-depth learning with active learning methods that encourage participation and taking responsibility, direct the student to think and make inferences, and share ideas. [55], [56], [57].

It is stated that the learners who actively participate in the learning process - compared to the teaching processes in which they are left in a passive state- have a longer retention of the information they learn [58], [59], [60].

In this era and age, there is more need to have a student-centered mode of learning in which the student is an active participant in learning because the lecture method is not enough to ensure effective learning [61]. In this context, the main role of the teacher is to make students realize that they are responsible for their own

learning, rather than transferring knowledge. To create a positive attitude towards mathematics, there is a need to organize learning activities that will make learners enjoy and appreciate mathematics, illustrating the application areas of mathematics, focusing on the role of mathematics in building reasoning and critical thinking abilities, and offering the learners a chance to feel success in solving problems [62]. The methods called active teaching methods include techniques such as short animations, group discussions, problem solving and role playing [63]. In the teaching-learning process, in which active teaching methods are used, the participation of the students increases and their motivation increases.

In a classroom environment, the learners should be given a chance to actively take part in the learning process because in this way they are more likely and willing to engage and seek more information that is significant and essential for them. The class should therefore be planned and organized in this way to promote a student-centred approach of learning where they can talk, engage and write about what they have acquired and even apply it in their daily lives. [64].

In recent years, with the decrease in students' mathematics achievement, alternative methods have been started to be used in mathematics teaching in order to enable students to actively participate in the learning environment. In mathematics teaching, new alternative teaching methods and applications are being developed instead of methods that have been going on for years and can no longer be productive. Accordingly, in many countries in mathematics teaching; There are studies on alternative learning methods such as information technology supported teaching, cooperative learning, learning with drama and games, learning with concept maps, learning through visualization, and problem solving [65]. In this context, the active teaching methods mentioned are presented below.

Today, the rapid development of computer-aided education tools and the transfer of information to be transferred to the students in electronic environments necessitate the use of information technologies in newly developed educational environments. The computer, which is a product of technology, effectively presents information and responds quickly to requests, enabling it to be used as an educational tool. The computer can be used to gain concrete experiences learned in primary school, and to provide the connection and transition between concrete and abstract concepts in secondary school and high school. The computer provides materials based on visuality and discovery, contributes to learning with sound and images, can make learning activities permanent, enjoyable and productive under the guidance of teachers, and contributes to the active learning processes of students. Today, rapid changes in technology; The restructuring of mathematics courses and the updating of course contents have brought along innovative ways to learn mathematics. In this context, many studies show that mathematics teaching supported by information technologies gives much better results than teaching mathematics with direct instruction and contributes positively to students' learning processes and academic success [66], [67].

Learning mathematical concepts and operations is perceived as a chore by students; students see mathematics as rules for remembering when necessary, a series of difficult or meaningless formulas, and mixed methods [68]. Rapid developments in science and technology have affected the field of mathematics as well as in every field of science, students' interests, desires and needs have changed, and in this sense, the way the mathematics course is taught has also changed [69]. As a result of this situation; A contemporary understanding of mathematics teaching based on student effectiveness in the realization of creative thinking has emerged [70]. In addition to benefiting from information technologies, efforts are being made to make mathematics subjects easier, understandable and enjoyable with animations, stories and various activities. Among these, in addition to benefiting from information technologies, efforts are being made to revive the subjects and make them easier to understand and enjoyable with stories and various activities [71].

Active Learning

Active learning is a learning process in which the student is allowed to make decisions about the different areas within the learning process and is forced to make good use of their thinking and mental abilities during learning [72]. Active learning is based on the educational philosophy of pragmatism. Pragmatism forms the culmination of the tradition of child-or student-centered educational philosophy that began with Rousseau [73]. Active learning is a type of education in which students actively engage in the learning process by accumulating knowledge and understanding. Students will be able to blend multiple ideas and think creatively as a result of their increased understanding. Students must study hard and use new knowledge and expertise to acquire a deeper understanding through active learning [74]. 19. With the pragmatism that emerged in America in the century, concepts such as problem solving, practice, and experience began to be used. 20. Developed by Charles Peirce, will William James, and finally John Dewey at the beginning of the century, this system of philosophy adopts pluralism as a view of being because it falls more in line with the world and the nature of man. Pragmatism goes further than old inexperience and describes truth according to practical utility. The accuracy of something depends on it satisfying us, responding to practical benefit [75]. Learning is not about automatically emptying information into students' heads in a sequential manner. Learning: requires students' intellectual participation and application. Explanation and notation by itself do not provide long-term learning. Only active learning will provide this [76]. Active Learning, it has been expressed by various educational thinkers and writers together with the 20. Century and has been widely discussed with the argument that knowledge should be discovered by the student. Students' direct and active participation in the learning process is defined as active learning. Active Learning also means that in a course the student can participate in other activities besides listening and watching [11, p. 194]. Active teaching methods activate the thinking and cognitive skills of learners and this activity is maintained not only episodically but generally throughout the education process. The students are motivated to study and fully participate and engage throughout the lesson [77].

There are two main components of active learning. This means that the student is active and encouraged to think. According to C. Bonwell and J. A. Elison, active learning is when learners take part in the activity performed and thinks about what they do [63, p. 253]. Students strengthen their cognitive skills through active teaching. When working with any content, analyze and highlight what pupils have learned in accordance with the lesson's objective, and correct any ambiguities; Encourages students to communicate their thoughts and introduces them to their work [78]. The authors who are active in the field emphasize the need for students to work and be active in activities that include reading, writing, discussion, and problem-solving during learning [79]. Instead of buying, thinking, doing and Environmental Information configuration is targeted. In other words, when learning is active when active learning occurs, the learning-teaching process, teacher-student role ground will consist of:

• Students research possible learning goals and activities,

• The student chooses specific learning goals,

• The student is aware of which goals are chosen for which reasons,

• The student has self-confidence or develops self-confidence when necessary,

•The student makes the selection and program of the learning activities,

• The student has a learning motive or develops it himself,

•To start working on a topic of the student has its strategy,

•Student focus, remember what you've learned,

• Students read, listen and analyze what makes,

• The student establishes a relationship between the information, and, if possible, schematically shows the student to implement what they have learned to new situations,

•Explores new areas of application, and continuously check whether the student has learned,

• Understand that the student is holding on to what they have learned to understand the various ways that refer to,

• Students try to new learning strategies,

• The student explores the reasons for failure in case of failure,

• The student evaluates my performance,

•The student benefits from external sources for feedback on their performance,

•The student is motivated by thinking about the benefits of learning,

• The student manages his / her attention and energy well; he/she knows to decipher work where necessary,

• The students do most of the work,

• Learners spend most of their course time actively thinking, doing, and interacting with other students,

•The student uses their brains, they transform their ideas,

• The student solves problems and applies what they have learned,

• The active learning method is fast, fun, supportive, and attractive,

•The student often thinks away from his turn, moving and out loud,

• To this end, we first need to understand how learning takes place,

• Learning is not automatically emptying information into students ' heads in a sequential manner,

•Learning requires students ' intellectual participation and application,

•A student is constantly interacting with other students and with the teacher,

It is known that the courses, which are conducted according to traditional teaching methods, are insufficient for students to learn physics subjects. It is wrong to think that since traditional teaching methods do not contribute adequately to students' learning of basic physics concepts, they receive the best possible teaching. Before teaching, it is observed that the misconceptions that learners have about the concepts of physics are still going on or very little has changed at the end of the course and that the students are resisting the change. Students who easily answer problems that require numerical operations have a lot of difficulty with conceptual questions. It is thought that the students' success in tests involving numerical problems and their failure to work in conceptual tests is since there are many problems in traditional teaching methods. Many studies indicate that teaching, which consists of activities in which students, who are not active in teaching with traditional teaching methods, actively participate in the lesson, increases student success. In other words, it appears that active learning methods are more effective in learning some physics concepts and that these methods enable learners to be more active in classroom activities than in traditional teaching. The active learning method is a learning duration in which the learner is responsible for the learning duration, the learner is allowed to make decisions and self-regulate about various aspects of the learning duration, and the learner is forced to use his / her mental abilities during learning through complex untold tasks. Active learning can be described as "engaging in learning activities that give students considerable control over the learning process," according to [76, p. 126]. Active learning has been a popular area of learning, especially during the last twenty, thirty years, although it is not a new thought expressed by various researchers since the beginning of the twentieth century. The main reasons for this are the

changes in learning understanding after the 1970s, the need for Lifelong Learning due to the transition to the information age, the inadequacy of traditional learning, and the fact that active learning is more effective than other learning processes.

However, applying active learning methods in crowded classrooms is an ongoing problem. The first study for this problem was by the peer teaching method developed by E. Mazur [5, p. 17]. This method, now widely accepted, restructured traditional teaching with the use of short narration of the subject, followed by multiple-choice conceptual questions that students answer first individually then group.

Teacher and Student in Active Learning

These are the things that teachers must do for active learning to occur:

1. Students should ensure that they take responsibility for their learning.

2. They should get students to think.

3. Provide learners with a broader choice of education opportunities and techniques [80].

According to C. Meyers and T. B. Jones, the duty of the instructor in active learning is to spend less time as a presenter in the center, spend more time behind the scenes as a designer, as a squareographer in the learning process. In the active learning environment, the teacher should clearly state the course objectives and content, create a positive classroom atmosphere and have more information about their students [81].

Active learning, students, according to B. Harrison, T. Hudson and S. Williams

• have a personal interest in their studies,

• If they make choices regarding the results of their work,

• They test their own ideas,

• If they design and plan their own experiments,

• If they introduce their findings to the rest of the class,

• If they solve problems,

• Consult and socially interact with a purpose within groups,

• If they think deeply about their work and rearrange their ideas, it is formed [80, p. 310].

According to D. Bentley and M. Watts, active student,

• He can start his own activities and takes responsibility for his own learning,

• Can solve problems, make his / her own decisions,

• Can use what he learned to establish connections,

• Can organize himself and others,

• He can show his abilities in different ways,

• He does his work with great pleasure [82].

S. Tong expressed the active and passive student as follows; the active student collects information together by asking questions and getting answers about the world he / she lives in and creates a model for himself using this data set [83]. While a standard passive student listens to the teacher and collects information from his seat, the active student asks questions to the teacher and asks more advanced questions using the answers he / she has received.

M. Silberman makes the following recommendations to ensure student participation in an active learning environment:

a. Open Discussion: The practice of open debate is important for revealing students' views.

b. Answer Cards: The response to the question asked by the teacher is written on the cards dealt. Response cards both save time and are not a threat because they are anonymous.

c. Voting: The short survey prepared can be applied orally or in writing.

d. Subgroup Discussion: Students are divided into three or more subgroups. If there is enough time to discuss the topic, the subgroup discussion is very useful for all students to participate.

e. Learning Pairs: Enables spouses to learn from each other by ensuring the participation of everyone learning pairs can be easily used if there is not enough time for small group discussion but everyone's participation is desired.

f. Whipping: The teacher goes to each group and gets short answers to key questions. Whips can be used when we want to get a quick answer from every student. Example: What you want to do to stop global warming is?

g. Panels: Several participants are invited to present their views in front of the whole class. Panelists should be replaced to ensure participation.

h. Fishbowl (aquarium): While some of the students in the class form a discussion circle, the rest form an audience circle around this circle. It is a suitable technique for large group discussion. New circles can be added inside the circles, students can be changed to continue the discussion. Although it is time-consuming, it is the best method to combine small and large group discussions.

i. Games: We can use fun activities to reveal students' ideas or skills. The games provide maximum participation as well as help students remember points they might forget.

j. Identify the Next Speaker: Students may be asked to raise their hands for those who want to express their opinion, and then those who want to determine the next speaker [84].

C. Kyriacou, B. Manowe, G. Newson identified seven types of learning activities to be used in mathematics classes in the first phase of his research to examine the use of mathematics in high schools. One of these activities is related to traditional teaching and the other six are about active learning. In the second stage of the study, a questionnaire including which of these seven activities they would like to use was applied to the mathematics sections. Findings show that active learning is more preferred, but less of use in schools. In addition, the answers are; in recent years, it points out that there has been a great movement towards the use of active learning, especially in research-oriented tasks, small group discussions, computer aided teaching and long projects [85].

In a study conducted by H. Gür, a prospective mathematics teacher learning to teach mathematics using the active learning method was investigated. 12 PGCE mathematics teacher candidates in England and 57 senior teacher candidates in Balıkesir participated in this research. The results obtained at the end of the research study can be summarized as follows: How the teacher candidates in both institutions learned to teach, their attitudes towards teaching, their feelings and thoughts, the effect of mathematics they learned in middle and high school on their current learning [86].

It has been determined that their teacher education, pedagogical formation and internship practices, materials used in teaching, and teaching methods have an effect on teacher candidates' learning. In addition, it was determined that pre-service teachers who encountered active learning method in university education and learned to use it, quickly passed the steps in the ladder theory and reached the stage of reflecting what they learned.

M. L. Lununberg and M. Volman conducted a study to investigate students 'and teachers' perspectives on active learning in primary education. In the study, activities for active learning approach have been shown to students and teachers to gain experience. When teachers apply these methods, it has been observed by researchers that students exhibit passive behavior, take too much responsibility for dealing with students, and pay little attention to teaching them study techniques [87].

Keyser, W. Marcia compared active learning and cooperative learning and mentioned their effective use. When we look at the test; it has been proven that active learning techniques are applied more easily and are not as time-consuming as cooperative learning techniques, cooperative learning requires higher planning and may need to be applied throughout the entire term. In addition, it was emphasized

that the selection of the teaching technique should be made very carefully, and it should be appropriate to the level of the class and the goals of the lesson [88].

B. K. Berger did a research that entailed the evaluation and usage of active learning by graduate students at the University of Alabama. The research study showed that the usage of an active learning approach provides benefits to students in the fields of research and fieldwork, explaining what they have learned, and thinking [11, p. 195].

M. Nakiboğlu and M. Altıparmak, as a result of their study titled "Brainstorming as a Group Discussion Method in Active Learning", increased students' interest in the lesson, put forward with creative thinking how to use the information they learned, and developed an awareness of the necessity and importance of the information given to them. They have determined that they have developed their scientific thinking abilities, in short, to reach results by analyzing their knowledge and observations [89].

J. S. Rosenthal, in his study in order to apply and evaluate active learning strategies in higher level mathematics classes, stated that using alternative learning approaches including cooperative learning with small groups and essay writing tasks in technical subjects should be supported. Enhancement of the participation and interaction of students and improving their perspectives are emphasized. Findings obtained as a result of the research revealed that the application of various active learning methods positively affects students' learning [90].

S. Narlı compared the effect of active learning technique and traditional teaching method on learner success in teaching the subject of numerical equivalence, and the readiness of students studying at different universities or different faculties to the subject before teaching the subject of numerical equivalence with the success levels after teaching the subject of numerical equivalence. The numerical equivalence test was applied before and after the study and the results were compared with both groups. According to the outcomes, there was not found a difference between the groups in the pre-application. In the last application, although the groups improved within themselves, there was a meaningful difference in the test scores in favor of the treatment group. According to the outcomes of the open survey, there was no important difference in the views on "mathematics, mathematics department and abstract mathematics", while an important difference was found in favor of the treatment group in their opinions on numerical equivalence. In addition, it was observed that there was an important difference between the learners of Buca Faculty of Education secondary Mathematics Department and Secondary Education Mathematics Department in favor of secondary education mathematics students, in favor of Buca Education Faculty on the basis of faculties, and in favor of girls by gender [91].

A. Duatepe and B. Ubuz, in their study on the improving and implementation of a drama-based geometry lesson plan, the development and implementation of

lesson plans aiming to use drama in teaching and learning 7th grade geometry subjects are presented with examples. In addition, some experimental results are given briefly in this study. The research was conducted on 3 seventh grades in a common high school in the second semester of the 2002-2003 academic year. The sample of 34 students in each class consists of 102 students. Considering the class hours of the groups, two were assigned as treatment and one as control group. In the tratment group, the lessons were continued by the first researcher with lesson plans developed using the drama method. In the control group, it was carried out by a mathematics teacher using the method of direct instruction. In addition, Van Hiele's geometric thinking test (Z. Usiskin) to evaluate students' geometric thinking levels and mathematics attitude scale to measure their attitudes towards mathematics were applied to students before the main study [92]. In practice, after both units were completed, the access tests related to the units were applied. In addition to the two achievement tests, attitude scales and geometric thinking tests were applied again when the application was completed to determine attitude towards mathematics and geometry and the level of geometric thinking. Angles and polygons in favor of the group learning geometry with drama as a findings of the research; There was a statistically important difference between the scores of achievement and permanence tests, geometric thinking test of Van Hiele, mathematics and attitude scales of geometry on the subjects of and circle, circle and cylinder. These results support the findings that drama increases accessibility in different subject areas and supports remembering. These findings are also supported by face-to-face interviews. During the interviews, the learners in the treatment group mentioned that the drama-based geometry lessons were fun, permanent, demanding and intriguing [93].

Duran (2019) researched the academic achievements and retentions of students in active teaching, on the mathematics lesson Decimal Numbers. The sample of the study consisted of 71 6th class learners studying at Abdüllatif Şener high school in the Sarkişla district of Sivas province in the first semester of the 2016-2017 academic year. In the study, the lessons were taught with the treatment group with the peer teaching technique and the traditional technique in the control group for three weeks. Data was collected achievement test on decimal numbers, retention test. Achievement test was implemented as pretest and posttest and four weeks after the final test, a success test is implemented again to measure the retention of the learning. The findings were showed that active teaching method increases achievement also active teaching has more impact on retention than traditional education [94].

Cooperative Learning

Cooperative learning is a learning approach where learners offer each other assistance in an academic subject within a class by forming small mixed clusters in a classroom, and the success of the cluster is rewarded in different ways [95]. The positive effects of cooperative learning, especially in primary and secondary education levels, determined by research on academic achievements and other affective and social outcomes, have demonstrated that cooperative learning is an important variable in the learning environment [96].

The cooperative learning method entails taking part in a common cause with the intention of reaching a common goal. This learning technique entails pupils learning through communal engagement, or, to put it another way, "learn together." Students' interest in studying instructional material in the classroom is considerably increased when this technology is used [97].

Cooperative learning is a teaching technique that students work under the supervision of the instructor so as to achieve general learning goals in small groups. The general features of this method are:

1. Students are attached to each other to achieve the group's common learning goals.

2. There is a face-to-face supportive interaction within the group.

3. Students are assessed individually and each student in the group is held responsible for their sharing and contribution in achieving learning objectives.

4. Students develop appropriate collaboration and communication to help each other learn. In addition, each student presents their own experiences to the learning environment.

5. Students reflect and evaluate the effectiveness of group functionality for future learning [47, p. 34], [98], [99], [100].

The purpose of using cooperative learning; To improve the social and communication abilities of students, to increase the indulgence and academic success among students. With the collaborative learning, the researchers came to the conclusion that the students exhibit less competitive behavior in the classroom environment, cooperate more with each other, and develop the relationships between students with different characteristics. In addition, it is known that people learn best when they cooperate with others and play an active role in the learning environment themselves [47, p. 44], [101].

Cooperative learning is a teaching model backed up by many scholars such as Piaget, Carroll and Vygotsky. R. E. Slavin, stated that formal education carried out by adults is less effective in enhancing cognitive development than the child's teaching environment with his friends [102].

As J. Dewey, stated, experiences are some of the most significant aspects that play a role in the internalization and meaning of learning according to the progressive approach [98]. This can be shown as part of the most significant contributions of the cooperative learning method to the learning environment because, in mixed groups, students help each other to learn by presenting their different levels of experience to the learning environment during the activities. It is recommended that diverse learning opportunities for all groups be considered for the successful structuring of trainees' work in a cooperative form of teaching in a mathematics class. Each member of a group with varying levels of learning capacity, performance, and interests

complements the others [103]. This contributes to the realization of learning at a high level for a common purpose.

The cooperative learning method should not be compared with the traditional group studies currently carried out in our schools. Because there are important differences between traditional group studies and cooperative learning groups in terms of planning, implementation, and evaluation stages:

1. Positive interdependence among group members is an important factor in cooperative learning groups. The objectives are structured in such a way that learners are required to take care of all the cluster members other than their own competencies. The main element of cooperative learning is positive addiction. When a student needs help, one of their teammates helps him and the student is encouraged by his team or classmates to do the best he can [104].

2. There is a clear individual responsibility in cooperative learning groups. This responsibility is related to the material that every student will be evaluated and sufficient. Students give each other feedback on their level of progress. Thus, members of the group know who to help and who needs to be motivated. In traditional learning groups, students do not have enough individual responsibility to share in group work.

3. In traditional learning groups, it is generally composed of similar members. However, it is essential to create heterogeneous groups in cooperative learning.

4. In cooperative learning groups, all members share their responsibilities to perform leadership activities within the group. In contrast, a single leader is appointed in traditional groups and remains unchanged.

5. In cooperative learning groups, members carry each other's responsibility to learn. Group associates are expected to motivate and help each other so that they can continue working on them. In traditional learning groups, students rarely take responsibility for each other's learning.

6. In cooperative learning groups, it is aimed that each associate can learn at the highest level and to configure good working relations among the members. In traditional learning groups, students often work alone.

7. The social skills needed to work together in cooperative learning groups (leadership, communication ability, integrity against each other, resolving conflicts within the group) are taught directly. In traditional learning groups, interpersonal relationships and small group skills are often incorrectly formed.

8. When cooperative learning groups are used, the teacher; eyes the groups, help to solve the problems that arise when the students work together, gives feedback to each group on how to better direct the group works. In traditional learning groups, the teacher rarely helps groups and makes observations.

9. In cooperative learning activities, teachers are challenged by the clusters to configure the necessary processes throughout the study process so that they can participate more effectively in the learning process; in traditional group learning situations, instructors do not pay any attention to this [96, p. 27].

In cooperative learning, students develop the idea that when they work alone, they can achieve more than they learn. In collaborative learning groups, students work together, and besides those in the group, everyone is responsible for their own learning. Students encourage and support each other to maximize the learning of both themselves and other classmates in the group [47, p. 46]. When you put students in a group with different levels of learning together and give them a joint task while also determining each student's role, they face situations where they must participate not in their individual work, but in the work of the group, which most often leads to students' interest in joint collective work [105]. Cooperative learning is a student-centered approach and students learn actively. Instructors, on the other hand, take the role of facilitating learning rather than a teacher. When students work collaboratively, they have to present ideas, make plans and offer solutions to achieve their common goals. Thus, students develop socially and individually [106], [107].

S. R. Swing and P. L. Peterson, in their study examining the influences of learning environments on academic achievement, formed competitive, scientific and collaborative learning groups in science classes, and at the end of the study, they concluded that the most successful community was the collaborative learning group [108].

K.L. Whicker, M. Bol, and J. A. Nunnery, in their study comparing collaborative and individual learning in middle school mathematics lessons, concluded that cooperative learning is more influential than the individual approach in terms of accomplishment and social attitude [109].

In the study named "Effects of Traditional Teaching Methods and Cooperative Learning Method on Mathematics Teaching" conducted by E. Erçelebi, it was seen that there is a important difference between the cooperative learning technique and traditional teaching methods in favor of the cooperative learning method in terms of student accomplishment and retention levels of students. Moreover, it was found that the passive learners in the treatment group where the cooperative method was applied developed self-confidence when using the cooperative learning technique, that the students liked the mathematics lesson and started to be very interested in the mathematics lesson. The students wanted this technique to be applied in other lessons [110].

S. Akbuga (2009) aimed to determine the effect of teaching with group activities structured in accordance with the principles of cooperative learning in primary education fourth-grade mathematics teaching, with group studies not structured according to the cooperative learning technique, on learners' attitudes towards mathematics and accomplishment according to teaching. The research was

conducted by using a pretest-posttest model with a control group. The application was carried out with fourth-class students in a primary school in Izmir during the 2007-2008 academic year. "Achievement Test" and "Mathematics Lesson Attitude Scale" were used to collect the data. In the analysis of the data, arithmetic average, standard deviation, and t-test were used. Considering the results of the study, it is seen that there were meaningful differences in favor of the experimental group in terms of attitudes towards mathematics lessons and achievement levels between the experimental group in which group activities were used, which were structured in accordance with the principles of cooperative learning technique, and the control group, where group work was not structured in accordance with the principles of collaborative learning [111].

At the end of his study, R. E. Slavin found that thanks to the cooperative learning technique, students in the same class love each other more and that there is communication, dialogue, a sense of belonging to the group, and an effort to strive for a common goal [112].

R. Vhalery and Nofriansyah examined student activities in cooperative learning. Thanks to the cooperative learning technique in the research, it was found that silent students (rarely communicating or alienated) got used to communicating with other students according to the class; The students who did not dare to express their ideas started to express their opinions, the quiet classrooms became louder due to the "enlightenment" learning activity; students help each other, respect each other and their responsibilities are increasing; the distinction between rich and poor status among students has disappeared; It has been concluded that the relationship between instructors and learners is getting closer [113].

D. Hoek, J. Terwel, and P. Eeden investigated the effect of using social and cognitive strategies in the application of cooperative learning method on middle school students' mathematics success. In the study, in which 511 students participated in the pretest-posttest control group model, cooperative teaching was carried out in which social strategies were used in the first experiment group and cognitive strategies were used in the second experiment group. Based on the findings, the researchers stated that the incorporation of social and cognitive strategies in cooperative learning environments has positive results in mathematics teaching. In addition, it was stated that the learners with low achievement in the treatment groups performed better than the learners in the control group [114].

J. D. Nichols aimed to determine whether there would be a difference between the geometry achievements, goal orientations, self-efficacy, motivation and cognitive strategies of the treatment group students using the cooperative learning method and the control group students using the traditional teaching method. As a outcomes of the study, it was determined that the geometry lesson given with the cooperative learning method significantly affects the academic success, motivation, awareness of learning goals and self-efficacy of the learners. It was concluded that the correlation between self-efficacy and motivation was also at a high level [115].

H. M. Ahmadi applied his study in two different mathematics classes at Wisconsin Whitewater University. In the experimental group, he used an unconventional method of teaching and exploration. In this approach, he has adopted a cooperative learning approach in which students are active inside and outside the classroom. The study tested the effectiveness of the method used in terms of student motivation, interest, conceptual understanding and attitude variables. In the control group, he used the traditional teaching technique. After the analysis, he stated that student performances were better in the group where the cooperative approach was used, their attitudes improved, their interest in participating in outdoor activities, and their mathematics achievement increased [116].

In his study, K. F. Osterman found that students learned how to use language effectively, through the cooperative learning method, and that social interaction is more in the cooperative learning method [117].

In the study in which M. H. Matthews measured the attitudes of 800 students towards the cooperative learning method with the help of a questionnaire, it was concluded that the learners wanted the groups to be formed homogeneously [118].

C. Toumasis aimed to help students learn from books and contribute to the development of reading skills by designing various teaching strategies. They worked in collaborative learning groups with strategies designed with students. The students were given worksheets and materials to regulate reading and reading. A total of 100 learners from 8th, 9th and 10th grades participated in the study conducted in the mathematics course. As a result of the study, while learners were reluctant to read and study mathematics exercise books in traditional teaching methods, an increase in this desire and mathematical literacy skills was observed in students working with strategies in collaborative student teams [119].

R. Ravid and S. Shapiro found that students' success and communication skills increased in their studies in 4th, 5th and 6th grades using the collaborative learning method [120].

V. G. Carlan, R. Rubin, and B. M. Morgan investigated the effect of the cooperative learning method on students' mathematical problem-solving skills in a public primary school. The study was carried out with 5th-grade students during one academic year. In order to collect data, interviews were held with the students at the end of the application, and they were asked to write down their thoughts about their collaborative work and their effects on their mathematical skills. As a result of the research, it was observed that students were more willing to solve problems, they started to work in cooperation instead of competing, and they discovered that a problem has more than one solution. It was stated that students who generally do not want to work or do their duties are more willing in the problem-solving process. Students started to use more mathematical language and terms in their discussions in the group. The classroom teacher’s awareness of students' abilities has increased. At

the end of the study, the teacher transformed the classroom order from a sequential system to a group system [121].

M. A. Hossain and M. R. K. Ariffin aimed to compare the effects of structured cooperative learning, unstructured cooperative learning, and traditional teaching methods on mathematics accomplishment and attitudes towards the mathematics of secondary school students in Bangladesh. 105 learners took part in the study and the mathematics achievement and attitude tests towards mathematics were applied as a pretest and a posttest. The results of the study showed that structured cooperative learning has a significant effect on mathematics accomplishment and attitudes towards mathematics. In addition, it was found that structured collaborative learners performed better in mathematics achievement than non-collaborative and traditional students. Therefore, it was stated that structured cooperative learning can be applied to support learners' achievements in mathematics [122].

I. B. Karaoğlu's study in order to reveal the effects of traditional classroom teaching and collaborative learning activities on the accomplishment of fifth grade students in social studies course, their level of remembering what they have learned, and how classroom management processes are involved in classrooms where this method is applied, cooperative learning increases student success. The "Learning Together" technique is more effective than the traditional whole classroom teaching in terms of students remembering what they have learned or the retention of what has been learned that the cooperative learning technique is applied to the classroom and the classroom management processes in the classroom where traditional whole classroom teaching is applied. Found that there are important differences in favor of the classroom in which the learning is applied [123].

M. C. Mulryan conducted a research to examine students' passivity in cooperative learning groups in 6th-grade mathematics lessons. According to the outcomes of the research study, it was revealed that all students were more effective than other methods in their collaborative work and showed that they were more active in the lesson. However, it has been determined that slow learners do not benefit as much as fast learners. The reason the students remained passive was found to be that other students did not include them in activities. At the end of the study, it was emphasized that teachers who use cooperative learning should pay attention to students who learn slowly [124].

1.2 Psychological and Pedagogical Problems In High School Students

It is simpler to teach children what they like. Therefore, attitude towards mathematics is a psychological variable that should be considered in teaching mathematics [125]. The predominance of the closeness and friendliness aspects of the classmates draws the attention of children who are in search of support. It prevents them from getting bored in the classroom environment and enables them to concentrate more and find the mathematics lesson that they find difficult and

enjoyable [126]. At the same time, classmates have important effects on learning and teaching, especially in terms of their psychological effects, as places where closeness is used effectively [127]. Many researchers state that discussing with their friends is more effective than oral expression and emphasizes the use of active learning methods such as peer education in education [128]. In the lessons taught with the traditional teaching method, which is a teacher-centered teaching method, the student mostly chooses to memorize the subject. Students who actively engage their own feelings and thoughts in the lesson, along with their friends, are more likely to understand the subject [129]. Also, using peer teaching methods in learning reduces the sense of competition among students and creates a more supportive class atmosphere. Studies have shown that peer teaching in reading and mathematics increases success twice as much as computer-assisted education, and three times more in classes with a small class size [130]. Emotional processes are undeniable parts of learning [131]. Even if students forget the information they have learned about a subject, they do not forget their attitudes and tendencies towards that subject. M. Dereli concluded that teaching with classmates has a positive effect on students' mathematics achievement, their attitudes towards mathematics, and the permanence of the learned information, and also reduces their mathematics anxiety [126, p. 12]. W. K. Yoong conducted research to determine the extent to which peers affect the attitude towards mathematics and concluded that even people who hate mathematics can change their attitudes towards mathematics in a positive way, with the use of classmates in mathematics lessons, since peers are fun and enable comfortable thinking [132].

It is important to prepare an interactive learning-teaching environment in mathematics education. The teacher can provide in-class interaction through various activities during mathematics education. Among these activities, they can apply structured teaching techniques such as group work, play and discussion, which form the basis of learning by doing. Students can only interact by playing games, drama, discussing, doing and experiencing, and it can be easier for them to learn mathematics. In studies L. Huetinck and SN. Munshin [133], S. Olkun and Z. Toluk [134] it is stated that activity-based mathematics teaching makes students more productive and active, and learning by doing is effective in developing positive attitudes and behaviors towards the mathematics lesson.

Mathematics is a fun game as well as serious business. If mathematics comes first in the list of the most disliked subjects during and after primary education, the reason for this lies in trying to teach mathematics to the child without taking this question into account. Instead, the teacher should enter their world and seek ways to embody mathematics and make it enjoyable. Since the main thing for the child is to enjoy learning, mathematics should be a game for him in the beginning. Discussion with classmates both makes students active and make teaching effective. For this reason, it is thought that working with peers is a good way to popularize mathematics and an effective method to teach it [134]. When the recent studies in this field in the literature are examined, it can be seen that in the studies of S. Yazıcıoğlu and S. Çavuş-Güngören [135], F. A. Akın and B. Aıcı [136], N. C. Aksoy [137], M. T.

Sönmez and P. Dinç [138], Z. Kablan [139] peer instruction learning and lecture method are used. The success status of the classroom environments based on the study was compared. When the results are examined, it is seen that the student success in the classroom in which the peer teaching method used environment is higher than the student success in the traditional classroom environment.

A. Duatepe and O. Akkuş stated that teaching with active teaching methods will enable students to teach mathematical concepts, allow them to share mathematical ideas in the classroom, and enable them to learn mathematics by understanding and associating it [140]. A. Duatepe, Paksu and B.Ubuz stated that active teaching methods facilitate learning and provide a learning environment based on communication and cooperation [141]. They also emphasized that active teaching methods attract students' attention, create, andotivating and interesting learning environment, and thus develop a positive attitude in students. D. W. Haylock and S. Öztürk Karataş state that creativity has an important place in children's doing mathematics, and that teaching based on creative thinking has a positive effect on the child's problem solving skills and creative thinking level [142], [143]. N. Tekerek and S. A. Henkel, on the other hand, stated that creative peer discussions play a role in the development of entertaining creative ideas for students in mutual trust, sincerity and cooperation in appropriate places, in the company of expert, creative and well-equipped leaders, with a new understanding that will unleash one's energy and creativity and they say it's useful [144], [145].

When the causes of the problems encountered in mathematics teaching in our age are investigated, it is stated that these are mostly caused by traditional teaching.

Today, where the necessity of raising students as individuals who think logically and creatively, question, research, solve problems and take responsibility for their own learning is emphasized by all education researchers, the weaknesses of traditional teaching are listed as follows.

In the teaching approach that focuses on transferring knowledge, the teacher is seen as the only authority in the classroom. Teaching methods dominate, in which students accept the ready-made information without questioning them, and where interpretation, personal opinions and creative thoughts are not included. The over-dependence on textbooks makes the individual differences between learners and their learning needs not be considered. In addition, they are not motivated to research and don't push themselves to seek more information. When being tested, they write the information already shared with them without any extra knowledge or comment. Interaction and information exchange in the classroom is very limited. Therefore, it also slows down the development of students' social aspects. Pedagogical supervision and control should occur invisibly from the outside and only when pupils are unable to complete the job or locate the correct solution. In group exercises, the instructor should pay particular attention to what pupils do well and what they do poorly, and

then, if required, sort out any issues that occur [146]. In the traditional teaching meyhod The student is directed not to study but to memorization. The student does not question the information he has acquired, and does not investigate the reason. It is very difficult to motivate the student who attends the lesson as a passive listener, to attract his interest in the lesson and to keep his attention for a long time.

In the study of M. Coşkun and M. Güçlü, it was concluded that teaching with the active participation of students, such as cooperation, is effective in increasing the success levels of students [147], [148]. This indicates the necessity and importance of method change in teaching. On the other hand, only one of the students considered the education system as a solution to the problems. It can be said that this situation is due to the fact that students do not fully recognize the concept of the education system. When the solution proposals brought to the problems encountered in the mathematics learning process are considered according to the opinions of the teachers, in a way that overlaps with the opinions of the students; teachers are looking for the solution themselves. In the study conducted by M. Ünal, it was stated that the problems would disappear when the teachers developed appropriate methods, made the students love the lesson and encouraged them [149]. In the study conducted by Z. Bayrakdar Çiftçi, L. Akgün, and D. Deniz, which supports this finding, it is emphasized that teachers can be the focus of solution in the mathematics learning process and it is claimed that teachers should always take an active role [150].

In the majority of the current studies conducted, the learners who are taught using the active learning technique become more successful in academics and develop a more positive attitude than students taught by direct methods in their lessons. This study also indicates the same findings, with students from active learning classrooms having a higher mathematics achievement than students in the lecture method classrooms. Hence, as societies change into the age of information, instructors and teachers who are the enlighteners should benefit from the active learning methods and techniques in the education process even at the highest levels, fulfilling the needs of the modern era to increase the quality of learning in general [151].

All concepts in mathematics are related to each other, each new concept is another relationship built on the previous concept. Today, it is accepted that an effective learning in accordance with the structure of mathematics can be achieved with "relational learning" [152]. Relational learning consists of conceptual and operational knowledge and the link between them. After gaining conceptual and operational knowledge, the student cannot learn mathematics if he has not been able to establish the link between conceptual knowledge and operational knowledge. One of the most important goals of mathematics teaching is to enable students to learn mathematical concepts and abstract information correctly and to relate these concepts to their previous knowledge in a meaningful way. Establishing a relationship between concepts in mathematics shows that concepts and relationships are learned. Concepts and relationships in mathematics do not mean mathematically when used alone. Concept maps consist of rectangular boxes or circles arranged hierarchically. Two or

more concepts enclosed in boxes are associated with each other in short sentences with the help of arrows. Thus, concept maps visualize knowledge, concepts and relationships between concepts by arranging them hierarchically. A concept map is a graphical presentation consisting of concepts and conjunctions that provide the relationship between concepts [153]. Concept maps not only facilitate meaningful learning, but also transform students from passive listeners into active learners [154]. Concept maps provide long-term learning of information, reduce the retention of meaningless information, and transfer knowledge for future problem-solving activities [155]. The approach that organizes the concepts according to their hierarchical relations is an active, creative, visual and spatial learning activity. Students combine related concepts related to a topic. This spatial representation of concepts leads to meaningful learning [156].

One of the important components of the learning-teaching process is teaching materials. Teaching materials make it easy for students to learn the subjects. The selected material, in addition to meeting the gains of the curriculum; It should also have features such as ensuring the student's active participation in the lesson, arousing curiosity in the student, and being technically usable. Studies supported with materials in classroom environments are very important in developing students' critical and creative thinking skills. Technology can offer important opportunities in order to concretize abstract subjects and concepts in mathematics and to reach generalizations by establishing relations between mathematical objects. An alternative to concretize abstract topics and concepts is worksheets. Worksheets are defined as “teaching materials that are prepared considering the subject/unit gains, have the potential to be used in the education process, have explanations on them, and can be used by students in in-class and extra-curricular activities” [157]. Worksheets, which are one of the materials that help the implementation of activities suitable for acquisitions in constructive learning environments in the classroom environment, are tools that show students what to do in the form of process steps and allow students to construct the information in their own minds[158]. It is stated that these tools make students more active by ensuring their active participation in the lesson, helping students to construct the knowledge in their minds by providing a better understanding of the lesson [159]. As a result of the researches, the worksheets; It helps the educators to reach the students to the concept and to determine the learning level of the students and the effectiveness of the teaching, increase the students' interest in the lesson, enable them to be responsible for their own learning, make the necessary connections, construct the concepts in their minds, perform effective concept teaching, eliminate misconceptions and increase success H. Ardahan and Y. Ersoy [160], provides evaluation at the end of the teaching process A. Ceylan, E. Türnüklü, and S. Moralı [161]; It is stated that it makes learning enjoyable and making it a habit to draw conclusions S. Kurt and A. Akdeniz [162].

1.3 Didactical Principles of Mathematics Lesson Using Peer Instruction

The most common understanding of the method in classical mathematics didactics is as an ordered set of didactic techniques and means by which the goals of classroom instruction, upbringing, and development of students are realized at a specific phase of learning, transforming from teaching goals to learning outcomes[163].

The suggestion of the National Council of Teachers of Mathematics (NCTM), in the book “Principles and Standards for School Mathematics”, that instructors should use a more student-centered approach in the education of mathematics [164]. Defined as one of the student-centered active learning techniques, peer teaching is seen as one of the learning methods that increases the individual participation of the student in the lesson, enables students to learn by directing them to discussions with their friends, and where the student takes the greatest responsibility for their own learning [165].

Peer instruction as a concept first appears in the content of the exemplary working model developed by Hungerland for office environments in 1973 (A. G. Şekercioğlu Çirkinoğlu, [166]; A. Yaşar [167]; K. Yayla, T. Yayla, & O. Şimşek [168]). J. E. Hungerland explained the working model he developed as “modernizing” office environments. In this model, peer education was used systematically, and it was ensured that education was carried out with low risk without the need for any other teacher and teaching material [169]. In summary, in the peer education that J. E. Hungerland presented as a model, first of all, individuals whose applications are received are subjected to the placement test. Later, the student starts training as an intern, learning and mastering on the job and through peer education [169, p. 12]. In the peer teaching model suggested by H. Bialek, J. Taylor, and R. Hauke students have the opportunity to apply what they learn [170]. Thus, learners can learn at the highest level from what they learn by doing and experiencing. On the other hand, in the peer teaching method, the learner's feeling that he has to teach his friend causes the student to pay more attention to what he has learned and thus increase the sense of responsibility he assumes [168, p. 1746]. In short, in this model, students are responsible for both learning in the best way and teaching what they learned to other students in the best way.

The peer teaching model used by E. Mazur [5, p. 16], J. E. Hungerland [16p, p. 13] and H. Bialek, J. Taylor, and R. Hauke [170, p. 21] was developed for use in physics lessons in higher education. Here, the peer teaching method is expressed as students' working in groups of two or three, not alone, during the course. According to E. Mazur, the principal purpose of this application is to draw the attention of learner on the determined concepts and to benefit from the interaction of students with each other in the group during the lesson [6, p. 17].

E. Mazur developed the method of peer teaching based on his experiences and applications in physics courses taught by him at Harvard University. Peer instruction is a teaching method in which students’ think about conceptual questions and contribute to their learning by discussing them with each other, while the teacher gives the key concepts and guides the lesson more [5, p. 7].

E. Mazur states that in the Physics Department of Harvard University, where traditionally the course is taught by explaining and solving questions, students are unable to answer physics questions on a conceptual level, even if they solve mathematical questions [5, p. 11]. As a result of investigations on learning how different issues in the same way that physics students learn algebraic problem solving successfully the traditional teaching method students to grasp even the basic concepts of physics emphasized to the point that the benefit of enough [6, p. 16].

Peer instruction method is a teaching approach used to involve students' interest in the classroom through a designed question process to cover every student [171].

In addition, according to J. Latulippe, it was observed that the attitudes, trust, beliefs and expectations of students who are taught with peer education increased more positively than those who were taught with traditional education [172].

The peer teaching method is a method that facilitates the interactive and active participation of students within crowded classrooms [173]. This course aims to teach students the problems and concepts related to the subject with the help of discussion with their peers [6, p. 14]. T**.** Gok expressed that peer instruction cheers learners to be accountable for their studies and stresses on comprehension. It is not a refusal of the lecture shape, but a better option for learners who learn through various methods [174].

The interaction of each student and the mutual questioning of each other's concepts and their concepts make peer teaching effective. Besides, the way courses are handled makes the course attractive to students. The learners have the opportunity to discuss and compare their ideas with their classmates. Thus, concepts are restructured [5, p. 3], [174].

The most important feature of the peer teaching method is that students supplement the concepts by discussing the basic concepts in the group. For the students to discuss each other and produce an idea, the students must also have a basic knowledge and a preliminary preparation. Students can only realize this information by synthesizing the short presentations that the teacher tells in the lesson together with the preliminary preparation before coming to class [5, p. 9].

Peer teaching is a teaching method that aims at conceptual learning and to keep the student cheered up; the courses are divided into small parts/concepts and then processed in such a way as to allow evaluating with short conceptual questions. Active learning method courses are handled differently from the traditional question-and-answer method. It is aimed to ensure that all learners are active in the course by discussing conceptual questions in small groups. Teaching environments where this method is used; students ' discussions to convince their peers to remove the lessons from monotony, students are forced to think through concept questions, and learners are encouraged to use the knowledge instead of the presentation of readily available information.

Application of Peer Instruction to Mathematics

Traditional teaching methods are curriculum-based teacher-centered methods. These methods advocate that knowledge and skills should be taught and taught directly by the teacher. However, more student-centered methods advocate that knowledge and skills can only be gained by the student's activities. The aim of the new methods adopted in mathematics; to educate individuals, who can utilize mathematics in life, can solve problems, express their theories and solutions, can work teamwork, have self-confidence in mathematics and acquire a better attitude for mathematics [175].

Students tend to learn what interests them and what they consider as important to them; therefore, it is necessary to use innovative techniques that activate the student instead of the traditional methods in which the students are passive in the teaching and learning activities in the classroom.

One of the most important methods that make the student active is group work. It has been shown that the students' critical thinking and problem-solving skills have been developed through group work, their skills of expression have come to the fore and they have developed a positive attitude towards the subject. Through group work, students are actively involved in the training period and effective learning is realized.

It is seen that group work in mathematics teaching has an important place in mathematics learning because it provides an environment in which NCTM (National Council of Mathematics Teachers) 1989 report asked questions, discussed ideas, heard listening, had responsibility, made constructive criticism and formed mathematical knowledge [164, p. 57].

Peer instruction, which is used mostly in the health and guidance field in our country, has been applied only in science courses, and it has been concluded that it affects the success and attitude positively. Our aim in the present research study; to explore the effects of peer teaching based on group work on success, attitude.

Mathematics is difficult for students because it consists of abstract concepts. The worksheets used in peer instruction, visual materials, group studies, concrete activities provide students with a better understanding of the subject. Teachers and students take different roles in peer instruction.

Some roles of the teacher: The teacher is self-developing, directing, motivating, developing and practicing, questioning, questioning, suggesting, arguing, listening, working together, and evaluating.

Some roles of the student: The student is a physical and mentally active participant in the learning process, who is responsible for learning, who is speaking, asking questions, questioning, thinking, discussing, understanding, solving problems and working together, and evaluating.

As a result, in peer instruction, students can discuss the solution of a problem with their group friends and develop different solutions to the problem. In group work, students support, encourage, and value each other. This allows learners to increase their self-confidence. It can be used effectively in a peer instruction mathematics course, which makes the social interaction between students and communication easier by making communication easier.

The Role of Teacher and Student in Peer Instruction

For effective peer instruction to be carried out the stages of application should be well programmed and every stage should be monitored. For this reason, the teacher has more duties than the learner. In the peer instruction process, teachers become models of how to teach students to help and create opportunities for each teaching step. The instructor has a great responsibility for the selection of multiple-choice concept test questions, short lecture time, program execution, monitoring of the students' response times, and final explanation of the correct solution. Also, the teacher is should pay attention to:

1) Set clear goals for each session,

2) Select individual activities and comprehensive materials to achieve the goal,

3) Present the material and note responses, use feedback and consolidation,

4) The student should be guided to understand the teaching model by peer and to work with a peer,

5) Create a competition between the student and his / her peer,

6) Arrange sessions not longer than 30 minutes,

7) Observe and evaluate the teaching model with peer periodically. Peers and students should be given feedback,

8) Give information about the education of peers in the families and direct them in a way to support them,

9) Consider the special needs of the students.

In peer instruction, the teacher forms groups according to the students' levels of achievement. The student who will take the role of the instructor will work before the lesson. Since the student will assume the role of the teacher, both self-confidence and sense of responsibility develop. The student's task is to tell the group friend and then to solve the questions in the worksheet. The student should be given clues instead of telling the answer. Thus, the student reaches the solution itself. Since students can ask each other more comfortably about the places they do not understand, this study benefits both the students and the students.

Positive Characteristics of Peer Instruction

Student-centered activities increase the success of students. The studies emphasized that students should not be dependent on the teacher or the book and should be encouraged to discuss with their peers. Peer instruction increases the understanding and engagement of the learners regardless of their background information [5, p. 23], [6, p. 10], [176]. Peer instruction raises students' conceptual comprehension, reduces failure rates, improves learner attendance, and supports learner engagement and attitudes to their course (E. Mazur [5, p.9]; Lucas [175, p. 222]; L. Porter, C. Bailey-Lee, & B. Simon [177]; W. Beekes [178], L. Deslauriers, E. Schelew, & C. Wieman [179]; B. Noonan, & C. R. Duncan [180]). Peer instruction supply a process of reasoning during class discussions, permits participants to challenge each other with discussions, and enhances peers interaction (D. J. Nicol, & J. T. Boyle [21, p. 460]; N. Lasry, E. Mazur, & J. Watkins [181]; J. K. Knight, S. B. Wise, & K. M. Southard [182]), According to T. Gok Peer instruction enhances students' skills to solve problems and profit new comprehensions as an outcome of the thinking process[183], T. Gok [174, p. 421] reported peer instruction decreases learners' number who drops out of the course and Peer instruction reduces the gender gap in learners' conceptive learning (C. H. Crouch, & E. Mazur [6, p. 15]; F. Demirel [15, p. 70]; T. Gok [183,p. 23]; M. Lorenzo C. H. Crouch, & E. Mazur [184]; Miller, et al., [185]). D. Campbell and I. Erdogan, state that students are more motivated and more confident when they work with their peers [186]. Also, it was revealed that peer instruction positively improved the self-esteem and communication skills of young people S. E. Robinson, S. Morrow, T. Kigin, and M. Lindeman [24, p. 38], increasing their empathy skills G. A. Martin, and J. M. Double [187], supporting their academic development and personal achievement [188]. B. Schmidt, as a result of his study, stated that the peer education method increased students' satisfaction [189]. Benefits of peer instruction have been observed over many disciplines, including astronomy; P. J. Green [190], biology; J.K. Knight, S. B. Wise, and K.M. Southard [182], H. N. McKnight [191], M. K. Smith, W. B. Wood, K. Krauter, and J. K. Knight [192], calculus; M. Cronhjort, L. Filipsson, and W. Weurlander [193], E. P. Ferreira, S. Nicola, and I. Figueiredo [194], A. Lucas [175]; S. Pilzer [195], chemistry; M. F. Golde, C. L. Koeske, and R. McCreary [196], O. Ozcan [197]; T. Yıldırım and N. Canpolat [198], computer science; R. Caceffo, G. Gama, and R. Azevedo [199]; D. Zingaro, and L. Porter [200], L. Porter, C. Bailey-Lee, and B. Simon [177, p. 178], physics; C. H. Crouch, and E. Mazur [6, p. 9], Eryilmaz [15, p. 4], E. Mazur [5, p. 12], Gok [4, p.69], physiology; R. N. Cortright, H. L. Collins, and S. E. DiCarlo [201], M. J. Giuliodori, H. L. Lujan and S. E. DiCarlo [202], J. Michael [203], S. P. Rao and S. E. DiCarlo [204].

Other benefits of peer instruction:

1) Peers talk, discuss and learn more easily among themselves,

2) Acquires the ability to be more independent in the face of the authorities through the peer group,

3) Students can complete a learning task with their steps without comparing with the faster learners,

4) Peers are entertaining, not threatening to their friends,

5) The angle of view of the young person in the peer group,

6) New behaviors are gained by identification in peer groups,

7) The knowledge and skills gained are useful for the young adult's life,

8) Provides leadership experience to young people with leadership skills

9) Develops cooperation and team spirit learning,

10) Causes young people to take responsibility,

11) A cooperative learning relationship is established with the feeling that he/she has equal status in the peer group,

12) Peers try to help each other in collaborative learning,

13) Peer trainers understand the problems of other students of the same age because of their cognitive characteristics.

Problems in Peer instruction

Although it is seen that peer counseling programs have started to increase in our country, it is seen that very few scientific studies have been conducted in this regard. Therefore, we do not have detailed information about peer education.

The problems encountered in peer instruction;

1) The lack of clear objectives and objectives for the program,

2) Time: Some learners require more time to think for multiple-choice concept test questions; hence, teachers could not solve more multiple-choice concept test questions throughout a course,

3) The lack of a detailed curriculum for the training of peer instruction during peer instruction, the instructors should develop and outline multiple choice concept test questions linked to aspired educational goals and objectives,

4) Since we are unfamiliar with peer instruction, the studies carried out in education are not sufficient,

5) Peer discussion: Some learners do not like to discuss multiple choice concept test questions or taught subjects with classmates. Additionally, they may be bothered by their classmates when they answer an inaccurate response posed a multiple-choice concept test question; therefore, peer instruction might not reach the desirable grades,

6) The teachers have complexity in the learners' attendance, which might be difficult when discussing with their classmates in the lecture (B. J. Brooks & M. D. Koretsky [205], A. Lucas [175, p. 225], N. Michinov, J. Morice, & V. Ferriéres [206]). "Challenge is the difficulty in fully engaging learners in peer discussions." Since the teachers should motivate the participants on concept test questions, walk around the class during peer discussion, and support students to share their thoughts with classmates [207].

7) M. C. James and S. Willoughby observed that 38 percent of students' discussions between classmates were ordinary talks. The remaining proportion 62 percent of students' discussions were extraordinary talks. In this case, the teachers should preferably structure and organize peer discussions performed between participants [208].

In peer instruction, the selection of peer groups, training of peers, presentation of peer counseling services after education, evaluation of the education process, preparation of a suitable classroom environment to be given peer instruction, requires a lot of time, effort, energy, and most importantly strong teamwork. Therefore, peer instruction is not a practical approach that can be implemented immediately. However, it is an approach that can be yielded when the mentioned stages and standards are observed.

The Implementation of peer instruction

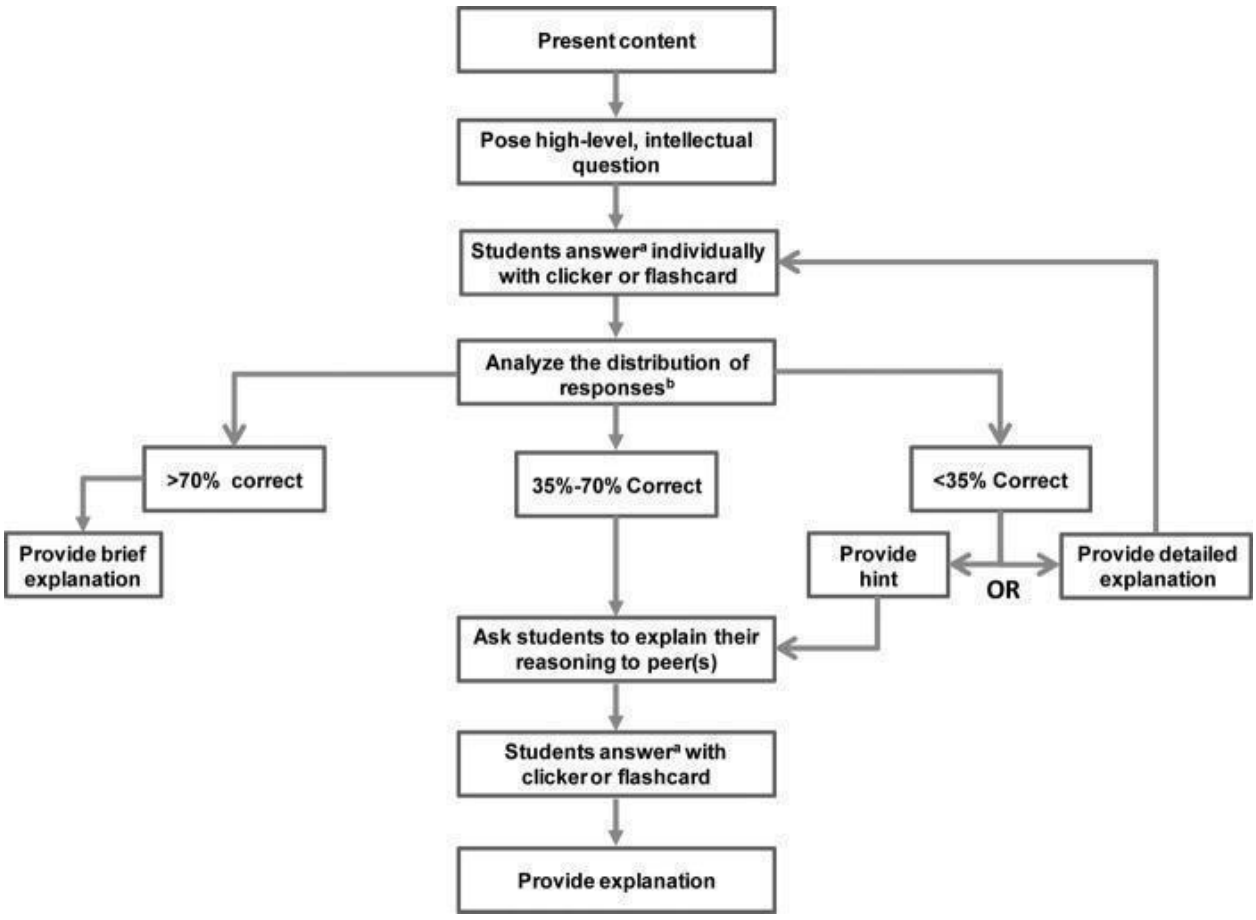
The application of the peer education technique includes seven steps. Initially, the instructor gives a short lecture on a concept in the course, it takes 15-20 minutes. Then, the instructor gives concept test questions; concept test questions are designed to evaluate the student comprehension of the basic concepts behind the lecture material. Students solve the question individually and give the first responses in 2-3 minutes. After that the instructor analyzes responses if the correct responses are less than 35%, the instructor explains the lecture again, if the accurate answers are between 35% and 70%, the class passes the discussion part and if the answers are higher than 70%, the instructor presents the next question. In the fourth step, students discuss their answers with classmates in 1-2 minutes. The previous studies (Catherine H. Crouch & E. Mazur, 2001 [6, p. 15], S. Kaymak [209], A. S. Podolner [210]) indicated that the discussion section is a significant part of peer instruction, affecting the participants' responses positively. After the discussion part, students give second responses. In the last step, the teacher collects answers and explains the question. In the response process, the general process of which is the adaptation of the think-pair-share technique, students can give their answers in different ways, sometimes they vote with colored cards or raise their hands instead of clickers [211].

C.H. Crouch, J. Watkins, A. P. Fagen, and E. Mazur define the concept test procedure as follows:

1. Concept test questions were given

2. Participants are given time to solve the concept test
3. Every participant records an answer
4. Participants discuss their answers with classmates
5. Participants present second responses
6. Instructor gathers answers
7. Instructor explains the solution [212].

Figure 1- This model could guide practitioners in an effective implementation of Peer Instruction (Vickrey T, et. al.,) [213]



In the application of the peer teaching method, both determining the percentage of correct answers and deciding how to continue the teacher can vary depending on the topics covered and the number of students [181, p. 1068].

Increasing the active engagement of students in a crowded classroom is one of the main aims of peer education [214]. This technique of peer learning encourages

peers to develop active and engaging discussions with each other, thereby, increasing their understanding and comprehension which later translates to their academic achievement in academics. What happens is, when a student does not understand the concept during the class after the deliverance of the short lecture by the instructor, he or she engages in peer discussions with another student whose comprehension was higher during the lesson. In this way, the students get help from their peers in areas where they could not understand at the end of the day and they are finally able to arrive at the correct answers to the conceptual test questions. [212, p. 57].

Concept Test

E. Mazur developed a concept test to teach physics as a part of the peer instruction method. Concept tests commonly used in physics class, have been successfully adopted and used in other disciplines (e.g., astronomy, biology, chemistry, mathematics) [6, p. 16]. To discuss and answer posed multiple choice concept test questions are quite important in the peer instruction technique. Peer instruction generally consists of three main parts: short lecture, conceptual test questions, and explanation of the concept question. According to I. D. Beatty, W. J. Leonard, W. J. Gerace, and R. J. Dufresne, the purpose of the concept test question promotes conceptual understanding and designed to address misconceptions in a particular content field [215]. To get the result of this purpose the questions should have a specific pedagogic aim on the other hand the difficulty level of the question may change. The correct level of complexity is the leading goal for an upper-quality question [216]. In the peer instruction, the participants' learning gains are raised when the difficulty of the concept test questions increase (Kaymak [209, p. 412], L. Porter, C. B. Lee, B. Simon, & D. Zingaro [217]; Smith et al., [218]) additionally J.K. Knight, S. B. Wise, & K.M. Southard, compared the difficulty of concept test questions designed with Bloom's taxonomy in a biology course and observed that with higher-order questions students' discussions became more sophisticated and students' learnings were increased [182]. On the other hand, the results of R. L. Miller, E. Santana-Vega, and M. S. Terrell, research show that, for some students, high difficulty questions and peer discussions may not result in higher performance in the most conceptual questions [19, p. 197]. However, a better comprehension of the concepts allows them to increase their accomplishment in the traditional sections of the course. It is seen in the comments of the students who are applied peer education that they want the concept test to be used by other educators [219].

Thinking time and First response

The choice of questions, the lecture time, the given time devoted to each question, and the number of questions should be adapted to the level of class and student [6, p. 11]. In Mazur's peer education model, the second and third steps are for students to think individually and implement their answers through voting. In several previous studies, the researchers have investigated whether or not students' time to think and respond to questions individually is required. For instance, D. J. Nicol and J. T. Boyle compared the two distinct implementations of peer instruction in their

study [21, p. 460]. Initially, they have applied the peer instruction with all steps to engineering students and in the second implementation they have applied the peer instruction without 2 and 3 steps. In the final of their study, the researchers reported that learners thought both methods improved their comprehension of the concepts. Of the learners who described their choice, %82 of the learners explained they choose to respond to the question separately before discussing it with their classmates. 80% of the class agreed that the individual reply time compelled them to consider and select a response to the concept test question; they observed that this guided them to be more effective and occupied during the discussion. K. L. Nielsen, G. Hansen, and J. B. Stav conducted a study. They found that the majority of learners thought that individual time was required to help them form their opinion without being affected by classmates. These studies indicated that students' engagement to a response before discussion enhances learners' learning and that steps 2 and 3 should not be bypassed during practice [220].

Discussion

The discussion part is the heart of peer instruction. In this process, the discussion between classmates improves the more profound thought, enhances complicated thought abilities on multiple-choice concept test questions, ensures to share and promotes alternate opinions and thoughts, locates different explanation methods [174, p. 641]. Peer discussion is the most well-known attribute of the peer instruction model, and most studies give knowledge on learning Achievements observed after learners' discussions. In the studies conducted, it was stated that the wrong answers given by the students to the questions after the peer discussion turned into the correct answer with a high rate. According to R. L. Miller, E. Santana-Vega, and M. S. Terrell, peer discussion affects the use of good questions [19, p.198]. The effect of discussions on more difficult questions positively influences students' responses to questions regarding the concept. (B. J. Brooks & M. D. Koretsky) [205, p. 1479], (A. D. Bruck & M. H. Towns) [221], (M. J. Giuliodori, H. L. Lujan and S. E. DiCarlo) [202, p.170], (S. Kaymak [209, p. 407]), (N. Lasry, E. Mazur, & J. Watkins [181, p. 1071]), (J. T. Morgan & C. Wakefield) [222], (L. Porter, C. B. Lee, B. Simon, & D. Zingaro) [217, p. 48], (Smith et al.) [218, p. 123], (A. M. Straw, E. Wicker, & N. G. Harper [219]), (J. G. Tullis & R. L. Goldstone) [223], (M. Willoughby, J. Kupersmidt, M. Voegler-Lee, & D. Bryant) [224] in their research they stated that after discussion, students' wrong answers changed drastically to right answers. S. P. Rao, and S. E. DiCarlo reported that the peer instruction method's effect depends on knowledge transmission from learners with accurate responses and with a common goal in their desire to achieve success to their neighbors during discussions [204, p. 53]. Also, N. Trottier, L. Kamp, and P. A. T. Mirenda concluded that the discussion process in peer education improves social interaction among students [225].

S. P. Rao, and S. E. DiCarlo, in a study they conducted with 256 first-year medical psychology students during 10 lessons, found that the peer teaching method significantly increased the rate of answering concept questions in the discussion

section. Three types of questions were asked in the study recall, intermediate and integrative. The rise of correct responses for recall questions were from %94 to %98, for intermediate questions from %82.5 to %99.1 and for integrative questions from 73.1 to 99.8. After the discussion part, the correct answers for higher-level intellectual questions were higher than the other type of questions [204, p. 54].

C. Y. Chou and P. H. Lin, in their study conducted at Yuan Ze University in Taiwan, one of the essential characteristics of peer instruction is the opportunity of students to randomly choose their discussion counterparts in relation to the effectiveness of the discussion process and the ability of the learners to engage in the discussion willingly. In determining the willingness of the students to participate during the discussion process, the instructor used a grading system during the group-formation stage in which the correct answers given by the group members affected the scores of the other students as well to a certain level. The grading system was 40% of the individual responses before the discussion, 30% of the individual responses after the discussion with their peers and 30% of the individual responses given by their peers after the discussion for groups made up of two students while 15% of the individual responses given by their peers in groups made up of three students. The study in which 86 students participated was conducted in a period of 11 weeks with the first 6-week stage incorporating pre-determined groups by the instructor in each of the lessons using the above grading system while the second 5-week stage involved random discussions by the students with their desk-mates in class. Moreover, the students received their scores only from their own answers that they had written down and did not receive any marks from the answers of their discussion peers after the consultation in class. The learners submitted to the instructor all their responses through an electronic answering system. In addition to this, they submitted an electronic self-evaluation report through the electronic answering system in which they mentioned whether or not they participated in the discussion and if it had any influence on their responses. It is also important to note that after the first 6-week stage, learners were asked to elaborate on their responses using a Likert-type scale containing 5 questions [226].

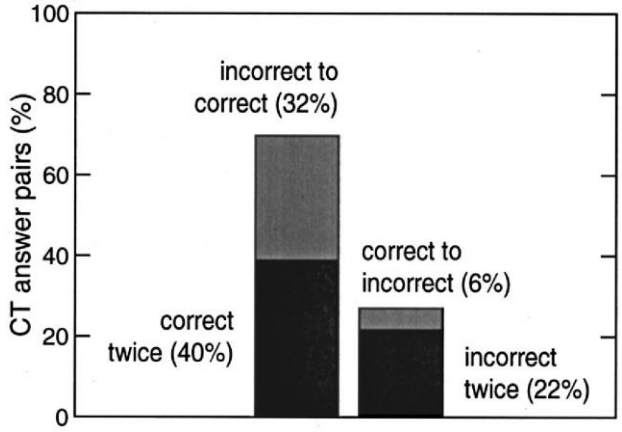
S. Kaymak researched the effect of the discussion section on peer instruction during the lecture. The application was carried out for five weeks with 30 students at Suleyman Demirel University. In the mathematics analysis course, 32 questions were asked, the average of the first correct responses was 16.625 and after discussion part the mean of the second correct responses was 26.625. The difference as a result of the analysis made with the independent t-test was found to be significant (p= .000). Can be said the discussion part increased that correct answers [209, p. 407].

According to A. S. Podolner, students' efforts to persuade their friends in the discussion section in the lessons taught by peer education increase both the rate of correct answers given to the questions and their confidence in the correct answer. While only 3% converts the correct answer to the wrong answer; it corrects the wrong answer that 29% of the students made first [210, p. 174].

Smith et al. (2009) investigated whether students were influenced by knowledgeable peers in the increase of correct answer rates after peer discussions during peer teaching in their study in the medical school genetics course in the USA. 350 students participated in the research study, and students were asked to have peer discussions by asking questions. After the discussion, the students were asked to answer them individually by asking a similar question measuring the same concept. As a result of the research, it was determined that peer discussion improved conceptual understanding and this result was valid for students who did not answer the question accurately at the first moment in the discussion group [218, p. 124].

Ten years of experience and results are the most extensive research on this subject, and the findings suggest that the debate is positive for students and has the most impact when the correct answers are between 35-70% at the end of the first answers [6, p. 13].

Figure 2 - Illustrates the alteration of students' responses that during discussion change from an inaccurate answer to an accurate answer (C.H. Crouch, J. Watkins, A. P. Fagen, and E. Mazur) [212, p. 48]



Also, many studies indicated that the discussion section influences the students' confidence [205, p. 1480], increases the students' conceptual understanding [227], impacts deep-learning of the students [228], improves learners' creative achievement and effective on development of their thoughts after assessment with peers [229]. Of the learners who described their choice, %90 of learners agreed that "a discussion with peers after an individual answer leads to deeper thinking about the subject." [21, p. 462].

Explanation

The instructor's explanation of the concept test questions also impacts the effectiveness of peer instruction [213, p. 5]. L. Porter, C. B. Lee, B. Simon, and D. Zingaro, Smith et al., 2011have published their study related to the influence of explanation at the end of the peer instruction process [217, p. 46-218, p. 123]. Smith et al., 2011 established three experimental situations; Peer discussion only, teacher explanation, peer discussion, and teacher explanation. According to the results, the third situation has significantly significant learning gains. Furthermore, these learning

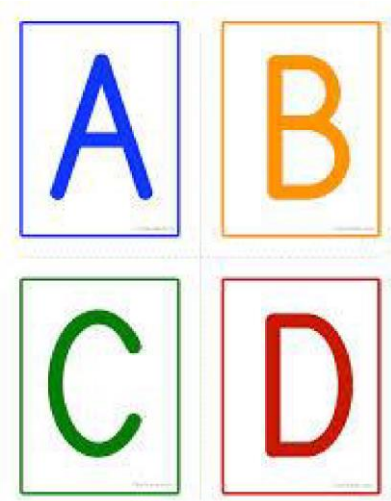
benefits were examined over both lessons and for learners at all levels of experience (low, medium, and high performing) determined by the average scores on the first question) [218, p. 125]. The first study of Zingaro and Porter, 2014 obtained similar results. Students tend to learn more effectively when taught by both the teacher and getting an additional explanation from their peers compared to just relying on their peers' explanation. [230]. Moreover, in questions that were more difficult, the instructor's explanation proved to be more essential to the students. L. Porter and D Zingaro in a different research discovered that the combination of classmates' discussion and teacher's explanation compared with classmates' discussion alone was positively related to performance on the final exam [231]. Furthermore, the outcomes of J. K. Knight, S. B. Wise, and K. M. Southard [182, p. 646]; Lucas, 2009 [1175, p. 230], C. Turpen and N. D. Finkelstein [232] showed that it is significant for teachers to discuss responses to concept tests questions with students and get into touch expectations for peer discussion clearly with a focus on sense-making.

Response systems

One of the large benefits of Peer teaching approach is that the ConcepTest questions answers give the teacher quick feedback on learner comprehension. Recording the responses can be achieved in a type of method, "Show of hands, Flashcards, Scanning forms, Classroom networks" [171, p. 46]. These evaluation methods have advantages and disadvantages of as explained below.

Show of hands: The learners are affected by other learners' responses during voting, and some of the learners are embarrassed by giving inaccurate answers by raising their hands. The teachers have difficulty collecting the participants' responses, and the instant feedback could take time in terms of the teachers.

Flashcards: The teachers could easily regard the flashcards and supply immediate feedback to the learners. The participants cannot readily see other answers because they raise their cards simultaneously, and the flashcards are single-sided. The masterdeficiency of this way is the lack of a persistent record.



Scanning forms: In this way given students scanning forms they note their responses to the Concept-Tests questions on these forms. The disadvantages are that it needs some work after each lesson and that there is lateness in feedback, the information being accessible only after the forms are scanned. Furthermore, participants may not deliver to approval, as they do not have to report their responses quickly.

Classroom networks: The major benefit of these systems is that correct outcomes are instantly accessible to the teacher, the participants cannot see the histogram, so their discussions are not affected by knowing which response was most commonly given by their classmates, and student knowledge is available to the teacher, making crowded classes more individual. The lack of classroom response systems is being more expensive than the other ways.

According to T. Gok, clickers were required in a crowded course environment for saving energy and time, and supply real-time feedback and flashcards were also

helpful in a not crowded course atmosphere [233, p. 73]. J. Latulippe reported that using clicker technology increase student participation [172, p. 610].

The advanced technological instruments receive formative feedback easier for lecturers to assess and interpret the answer of learners in a crowded classroom [181, p. 1067].

But on the other hand, N. Lasry, E. Mazur, & J. Watkins indicated in his study that the peer teaching method was an efficient teaching approach just and it does not depend on the utilize of advanced technological instruments such as classroom networks [181,p. 1068].

In a study on whether the use of clicker or flash cards in peer teaching practice would cause a difference in conceptual test success, no difference was found between the conceptual test scores of high-tech pointers and groups using low-technology flash cards, but it was mentioned that using pointers has some benefits. Students' answers to the conceptual test can be stored, a simultaneous feedback can be given about the question, and by using markers in peer teaching, it can respond to the need to change the focus from instructor to conceptual teaching [181, p. 1068]. Similarly, in a different study, it was found that there is no difference between pointers and flash cards [234].

In a study on peer teaching that requires the rapid identification of students 'correct answers, peer education was applied to 20 students in 60-minute periods with the pointer (i-clicker), it can be very efficient in terms of students' participation and learning, and thus students can see multiple approaches about solving questions. It has been expressed [168].

In another study, J. Eyink reported that the use of the clicker application at the University of Southern Indiana increased the participation of learners in classrooms in crowded classes, facilitated learning, and students were less stressed with 108 introductory psychology students [235].

A. Hoppenbrock, determined 100 undergraduate analysis course students as a sample in his study at Paderborn University in 2016. He examined the role of click questions in influencing collaborative discussions in the lesson. As a result, he encouraged the usage of quality peer discussions and the click questions because he discovered that they promoted the understanding and comprehension of students mainly in their conceptual understanding. He added that the usage of those click questions and discussions should be implemented more often in undergraduate courses [236].

Peer instruction and achievement in Mathematics:

Achievement is the competency displayed by students in their academic tests either teacher-made or standardized achievement tests administered by examination bodies. Achievement deals primarily with the success and academic performance of

students in these tests to ensure that they understood and can implement the learned concepts outside the class. Achievement tests generally measure the teacher's effectiveness in the learning process as well as the understanding of the learners; this means that they conclusively measure instruction and learning. A high achievement generally signifies understanding, accomplishment, and benefit in the learning process while a low achievement signifies weaknesses and a lack of attitude and understanding in the learning process.

Mathematics academic achievement has been a very big concern for Mathematics educators around the world, as a result, numerous research and studies have been conducted by academicians and instructors to address this growing concern. There is a general fear of Mathematics and a negative attitude towards it because of the teaching approach used by instructors hence this is reflected in the overall achievement of the students in their tests. Mathematics academic achievement relates to various other factors such as attitude and teaching method used.

In their study, K. Singh, M. Granville, and S. Dika concluded that attitude and interest affect academic achievement hence there is a need to develop new strategies that focus on improving students' attendance to lessons as well as active participation within the classrooms. The academic achievement shows to what extent the topic was understood by the learners and helps the teacher tell whether there is indeed progress in the education process or there's a need for reforms. Students who record a higher achievement are assumed to have not only understood the topic but able to apply it as well to real-life situations and come up with effective solutions while students with low achievement are assumed to have not understood the topic and cannot apply the concepts to other real-life situations [237].

Achievement tests used to test understanding differ from each other and have different structures depending on the instructor's choice of questions and outline. Some instructors use multiple-choice, others open-ended questions or incorporation of both systems. Attitude and interest are also tested using academic achievement because it is assumed a higher achievement in mathematics directly corresponds to a positive attitude and vice versa. K. Tarim and F. Akdeniz note that academic achievement tests help to evaluate success in various ways such as rewarding students by publicly acknowledging their effort and issuing certificates of success and as a result, this positively influences mathematics achievement and motivates the learners to improve [238].

M. Moenikia and A. Zahed-Babelan note that mathematics is a global subject that is essential to life and every individual is required to at least be familiar with the basic mathematical concepts and operations however mathematics academic achievement is ultimately influenced by the opportunities of the learners to learn [239]. Academic achievement is mostly limited to the hypothesis that high intellectual ability translates to academic success however H. E. Gruber He has concluded that students with high intellectual capacity do not necessarily grow up to

be creative grown-ups and that the skilled and creative grown-ups were not necessarily talented when they were young [240].

Mathematics achievement has also been closely related to self-efficacy and effective engagement in a class by various studies. N. Ozkal has found an important positive correlation between self-efficacy and achievement in mathematics. He writes in his study that although some other studies haven't found a relation between the two, in his research analysis, he found that learners with a higher self-efficacy belief for mathematics learning and performance had generally higher academic success compared to students with a lower self-efficacy for math learning [241].

Based on these studies, classes with higher academic-achieving students tend to be more supportive in terms of self-efficacy in math classes by using effective methods that promote active learning in the classroom environment. This puts students' feelings into consideration as well and ensures they have a positive experience during the learning process. As we can see from the above studies and analysis, although mathematics achievement has been researched, more studies need to be done on this area because the data is not enough to determine the main factor that influences achievement in mathematics. Some researchers have tried to see if there is a positive correlation between mathematics achievement and the incorporation of STEM education as well.

N. C. Siregar, R. Rosli, S. M. Maat and M. M. Capraro have found some promising statistically significant evidence to show that mathematics achievement is positively influenced by using STEM programs in education. They recommend teachers to utilize these STEM programs in their classrooms by using different instructional approaches to improve success in mathematics. It should also be noted that mathematics achievement involves enhancing student understanding and comprehension of the concepts in a way that enables them to easily apply the education and skills they learn in other areas as well [242].

Essentially, numerous learners get low scores in Mathematics. In like manner, the proportion of students perfoming poorly in mathematics to the total number of learners is a fundamental factor of Mathematics training quality. Most eminently, regarding mathematics instruction, it appears to be that it is hard for the students to adapt up to the topic due to the learners’ learning perspectives. Ordinarily, the students’ mentality in Mathematics is extremely negative towards the subject. Regardless of whether educators these days are truly receptive and cordial, yet the vast majority of the students fear and develop a poor attitude towards mathematics. This mentality compounds are now and again growing every year. Thinking about these perceptions, there is a requirement for quality instruction in the field of Mathematics of the instructors' that would assist them with exciting learners’ self-assurance, interest, and disposition for an intuitive class conversation. Along these lines, instructors need to decide the learners’ perspectives in understanding and learning Mathematics. This would fill in as a method of helping each learner enjoy the subject [243].

Numerous studies are showing that peer education improves the student's academic performance and knowledge. (F. Demirel [15, p. 82]; G. Akay [23, p. 75]; R. E. Abdelkarim & E. Abuiyada [244]; A. B. Lacaba, J. D. Magalona & T. V. G. Lacaba [245]; E. A. Oloo, S. N. Mutsotso, & E. N. Masibo [246]; Y. Z. Olpak, S. Baltaci, & M. Arican [247]; S. Ouko, C. Aurah, & M. Amadalo [248])

Peer instruction and attitude on Mathematics:

People interact with various situations they encounter throughout their lives. Permanent behavioral changes that occur as a result of this interaction are defined as "learning". Through learning, people gain knowledge, skills, attitudes and values [249]. Although the definition of attitude and behavior varies according to the area studied and the subject, it is generally accepted as a person's positive or negative attitude towards an object, situation or event. R. E. Petty and J. T. Cacioppo made a more comprehensive definition as follows: "Attitude and behavior are people's general evaluations about themselves, others or other objects, events or problems [250]. These general evaluations are based on many behavioral, sensory and cognitive bases and affect the development, change and formation in them." Attitude is a positive or negative intensity ranking and grading towards a psychological object [241]. Attitude is a sensory and mental preparedness that is the result of an individual's life and experiences, which has the power to direct or have a dynamic influence on his / her behavior towards all objects and situations it is related to [252], [253]. Attitude is described as being ready to react in a certain way towards a situation, person, thing, being towards, against or in favor of a concrete object or an abstract concept, and it is an indication of a person's understanding and feelings about a certain subject and It is defined as a feature that motivates to show a behavior [254].

Bloom, the resultant of the student's attitudes, interests, and the student's own knowledge finds affective input. In this respect, Bloom can use the student's attitude towards the course and school, academic self-concept and situations created by interests as affective input characteristics and points out the importance of these characteristics especially in terms of participating in the learning work [31, p. 77]. The students' visual input to learning and learning affects their success in school and the teaching situations they will encounter later. Success and failure in a course can change the quality of a student's feelings towards that lesson. The success and failure accumulated on each other also play a very effective role in the development of the student's academic self-concept. According to Bloom, approximately a fourth of the variable in academic achievement is dependent on the effective characteristics. Currently, mathematics is widely used in several other disciplines and activities but this trend is threatened by the declining achievement in mathematics. Looking at mathematics as a complicated and tedious subject is one of the major reasons why there is a noticeable decline in math achievement in schools. Mathematics education should not just involve stating the rules, definitions and methods for the learners to internalize rather it should involve the active participation of the learners by quality discussions and collaboration with one another [255].

In mathematics lessons, students generally stay away from mathematics activities, thinking that I will make mistakes. These students are mostly indifferent to mathematics class and do not like it. Therefore, as the mathematical activities in their classes increase, students develop a negative attitude. Here, great duties fall on the teacher and the family. An increase in mathematics achievement is only possible by breaking down this negative attitude. In order to develop a positive attitude towards mathematics, which is one of the most significant elements of mathematics education, it should be one of the duties of teachers and families to make students understand the importance of mathematics in daily life [256].

R. M. Capraro described attitude as “findings from a collection of measured experiences in the area of mathematics [257]. In the literature, most of the results indicated that there is an essential affirmative impact of peer instruction approach on attitudes towards the mathematics subject and the lessons (G. Akay [23, p. 47]; R. Abdelkarim, R. Abuiyada, & S. A. Siddiui [258]; J. B. Campit & R. M. Garin’s [14, p. 12]; F. Demirel [15, p. 92]; O. C. Yavuz [259]).

Related Research

A. B. Lacaba, J. D. Magalona & T. V. G. Lacaba aimed to investigate peer teaching as an intervention strategy that will increase the performance among 3-grade Mathematics pupils through the 1st and 4th grading periods of the education Year 2017 - 2018. The result showed a radical increase in Mean Percentage Score in mathematics subject after the intervention was given to the 3-grade pupils with the previous result of 74.22% from the 1st Grading into 82.11% MPS result in the 4th Grading term. The findings of the research revealed that the application of Peer instruction strategy has an affirmative impact on increasing the academic accomplishment in mathematics of Grade 3 pupils. There was a 7.89% rise for mathematics subjects from the first grading to the fourth grading period [245, p. 7].

F. Demirel investigated the effect of using peer instruction technique in a mathematic class on learners’ attitudes, performance and retention of learning. The study was carried out with 41 learners of two different primary schools in the 2011-2012 academic years. One of School was assigned as the treatment group with 20 students and the other School as the control group with 21 students. We did a statistical analysis on the "Peer Instruction” method that was implemented on the treatment group while the other “Traditional Instruction" was applied to the control group. The research techniques incorporated in the study involved both qualitative and quantitative methods with pretest and posttest conducted on both treatment and control groups during the study. We also chose very effective data collection and analysis tools such as the Mathematics Achievement Test, Mathematics Attitude Scale, and Retention Test. Survey conclusions have indicated that students in the experimental group have important improvements in their academic accomplishment towards mathematic lesson compared to the students in the control group. However, we couldn’t see any significant difference in their attitudes towards the mathematic lesson. It was observed that the peer instruction strategy has more influence on

learners' degree of permanence in mathematics in contrast with the traditional method. Additionally, data gathered from the research showed us that the gender of the students had no effects on success or attitude towards mathematics lessons. The students said that they liked math lessons much more and they were willing to take part in the lessons thanks to peer instruction. Their success was increased it was provided permanent learning with peer instruction. In addition, it was implied that this technique increased their responsibility, their self-confidence and it improved their friendship in terms of positive direction [15, p. 87].

M. Cronhjort, L. Filipsson, and W. Maria investigated the effect of the peer instruction method. Lectures were changed from the traditional method to the peer instruction method and were carried out 21 weeks with 2000 students on a calculus course. At the end of the implementation, the findings showed that peer instruction is an efficient teaching and learning method that helps students improve deeper conceptual comprehension [193, p. 106].

On the other study E. A. Oloo, S. N. Mutsotso, and E. N. Masibo proposed to indicate the influence of peer teaching on the performance of students in the teaching and learning process in mathematics. Their study was carried out in 12 randomly chosen schools in Bungoma with 167 participants. The investigation was contained Commercial Arithmetic, Circles, Quadratic Expressions, Vectors, Trigonometry, and Equations. Data was collected from students' questionnaires and students' achievement test. The study used SPSS and t-tests for data analysis. The result of the Students' Achievement test indicates that peer instruction strategy increases students' accomplishment in mathematics course. On the other hand, the conclusions indicated that peer instruction promotes learners' motivation to learn mathematics, improves comprehension of mathematical notions, and provides confidence in the learners [246, p. 12].

Y. Z. Olpak, S. Baltaci, and M. Arican researched the impacts of two distinct accountability scoring mechanisms on a sample of 46 learners from the 3rd course during the second period of the 2016-2017 educational years. This study was conducted on the topic of statistics and probability and made use of the peer instruction approach to observe the preservice secondary school mathematics educators' success in the aforementioned subjects. In the study for implementation, participants divided into two groups randomly, and the data were obtained using an academic success test and peer instruction and course assessment forms. And the result showed using accountability scoring mechanisms during peer instruction increasing students' success and providing learning activities [247, p. 2325].

S. Ouko, C. Aurah, and M. Amadalo investigated the effect of peer instruction on students' success in vectors. The implementation included four groups with 479 participants. The data was collected two achievement tests and for analysis t-test and ANNOVA were used. The results showed that the peer instruction method increased students' success in vector lesson more than the traditional teaching method [248, p. 177].

In the other study S. Awinoouko, the Role of Peer Teaching in Problem Solving Skills of Students' Problem Perceptions was investigated. Observation Learning Theory was used to guide the research. Previous Studies, indicate that using Peer Teaching enables students to acquire analysis, synthesis, and evaluation skills that facilitate problem-solving. The research study was conducted in Bungoma. Proportional Sampling was used to select 300 participants. The study used an actual research design after the research. A survey was used to gather data and analyzed by using both illustrative and inferential statistics. The results show that most of the students perceive themselves to solve problems in mathematics after peer teaching [260].

In another study, Y. Uesaka and E. Manalo explored the hypothesis that creating situations in which students must teach other students how to solve math word problems using diagrams would encourage students to use diagrams spontaneously afterward. Experiment classes 8. It was carried out in five days with 57 students in the class. All of the students in the experimental condition were allowed to explain ways to solve math problems given to other students in their group. In contrast, in the condition of control, only some students were allowed to make presentations about the way they solve problems in front of the class. In both cases, the teacher encouraged the use of diagrams during the given instructions. The original finding was that in post-teaching evaluation, those in experimental conditions had to prove that diagrams were spontaneously used more in their attempts to solve the given math word problems. These results suggest that as a result of peer instruction experience which provides an opportunity for the use of diagrams as a means of communication, participants internalized diagrams as tools of problem-solving. Peer interaction protocol has also been analyzed to better understand the mechanisms involved in this effect [261].

S. Pilzer published his study in 2001, and in the study was applied peer instruction over two semesters in a calculus class. He found an essential improvement in students' reasoning skills and mind-keeping skills, and based on the results, students responded approximately 90% correctly to conceptual problems. Besides, the attitude and confidence of the students positively improved toward the calculus course. Findings show that peer discussion allows thinking deeply, and it is efficient to use helpful questions. Also, the final exam outcomes showed that the peer instruction method had a substantial influence on all students and all groups [195, p. 187].

Different from other studies, E. P. Ferreira, S. Nicola, and I. Figueiredo analyzed the procedures and the results of the Peer Instruction method in an introductory Calculus course. The result of the study showed that peer education was successful in ensuring that low-level students were fully involved in the course. Most of the students reported that they were satisfied with the atmosphere of peer instruction [194, p. 106].

G. Akay, in her study, was published in 2011, examined the effect of the Peer Instruction method on mathematics performance and mathematics attitudes on the transformation geometry of 8th-grade students. The study consists of 112 8th-grade students of a state school. The two classes in which the researcher entered the course were randomly appointed as treatment and control groups. The participants in the treatment group were educated on the method of transformation geometry with the Peer Instruction method. Mathematics Achievement Test and Mathematics Attitude Scale were used as a measurement tool. The study finally concluded that by using the Peer Instruction strategy on the transformation geometry, there is an observable positive effect on the attitudes and the learners' achievement in mathematics after the lessons [23, p. 73].

J. B. Campit and R. M. Garin's in their research study used 30 2nd grade learners to observe the effect of the Peer Instruction method. The study was conducted to determine the effect of the method on the attitudes of the students during the second semester of the 2013-2014 academic year. More specifically, he has sought to identify and compare students' attitudes to mathematics before and after implementation of peer teaching approach and traditional teaching strategy. The experimental method was used especially in the pretest-final control group design. The data collection tool is the attitude scale performed by the current and reliable researcher. The weighted average, the Mann Whitney U test, and the Wilcoxon Signed-Rank test are statistical tools used in the analysis and interpretation of research data. The results indicated that there was an important alteration in the attitudes of the students in the peer learning group after being exposed to the peer learning strategy that was not observed among the learners in the traditional teaching group. Based on the findings, it was concluded that learners' attitudes towards mathematics were developed when they were exposed to Peer Instruction strategy [14, p. 12].

In their study, R. E. Abdelkarim and E. Abuiyada investigated the influences of peer instruction technique on mathematics academic success of bachelor learners in Oman. The study was carried out with 32 bachelor learners in the second period of the 2014-2015 academic years in "Mathematics for Social Sciences 1". The study was contained Properties of Linear Equations, Two variable systems of equations, Functions, Domain of functions and Properties of Exponential Functions. For the study, students were randomly divided into two groups. Data was collected with the Mathematics Achievement Test and analyzed using average, standard deviations, and independent t-test. The findings of the study show that the use of peer education increases students' success in mathematics lesson and peer education technique is an active tool to increase mathematical achievement [244, p. 126].

T. H. Allison investigated the effect of Peer Teaching based on the classroom performance system (equipped with CPS-infrared technology, an incredibly easy-to-use system that collects answers to questions from all students) on the academic success and motivation of 8th-grade mathematics learners. The control group study, which included 92 semi-experimental and non-equivalent 8th-grade students,

received traditional "classroom performance system-based" mathematics education. Again, 72 learners in the treatment group studying in the 8th grade were compared with the mathematics achievements of 92 students by using "class performance system-based Peer Teaching". Posttest scores were analyzed using ANCOVA. Basic Skill test amounts were used as a variable. A statistical control group design was used to explore learner motivation for the same group of learners under the same situations. Learner motivation data obtained through the "Instructional Materials Motivation Questionnaire" were statistically analyzed using MANOVA and independent samples t-test. As a result; It was observed that the mathematics achievement scores of the eighth grade students who received mathematics education using "Class Performance System-based Peer Teaching" were significantly higher than the learners who were taught by "traditional teaching based on classroom performance system without Peer Teaching". In addition, the student motivation scores obtained by the "Instructional Materials Motivation Questionnaire" were found to be significantly higher [13, p. 98].

In another study, R. Abdelkarim, R. Abuiyada, and S. A. Siddiui examined 32 female bachelor students' attitudes towards mathematics after the peer instruction method. The researchers conducted the study by dividing the students into two randomly selected groups. Data was collected by using Mathematics Attitude Survey which was applied to both groups before and after the process. Data were analyzed by using average, standard deviations, and Analysis of independent sample t-test. The results indicate that the participants after the peer instruction had a more positive perception of mathematics, which strongly influenced peer instruction on the attitude [258, p. 1503].

O. C. Yavuz researched the academic achievements and attitudes of students in peer teaching, which is carried out with a web-based peer and self-assessment system, on the mathematics lesson Rational Numbers. The sample of the study consisted of 472 learners studying at different schools in Keçiören district of Ankara province in the first period of the 2013-2014 academic year. In the study, the lessons were taught with the experimental group with the peer teaching method enriched with web-based peer and self-assessment, and the traditional method in the control group for nine weeks. Data was collected achievement test on rational numbers, Mathematics Attitude Test. Scales were applied as pretest and posttest. When the study is done, it was determined that academic achievement increased in favor of the treatment group, but there was no meaningful difference between the groups towards attitude towards the issue of rational numbers [259, p. 123].

A. Lucas was aimed to show that Peer Instruction and I-clickers improve learners attendance and comprehension. The study was carried out with 24 participants three times for 60 minutes a week in calculus lessons. The researcher separated the class time 10 minutes receiving and reviewing homework, 30 minutes class lecture, and 20 minutes Concept tests. The study outcomes indicated that using the Peer Instruction method with i-clickers enhances student participation and understanding [175, p. 221].

Peer Instruction on other disciplines

A comprehensive study of ten years has been carried out on the method of peer teaching by C. H. Crouch and E. Mazur. In this research, 8 years Peer Teaching Method and 2 years of traditional teaching method were applied. Approximately 100 students participated in the research every year and some years of lesson plans were revised and improved. The outcomes of this study examining the effectiveness of the peer teaching method; has revealed that students studying with this method understand the concepts of physics better than students studying with the traditional method. Besides, the peer teaching method has been found to improve learners' skill to solve mathematical problems. Researchers explain the development of learners' conceptual understanding based on their reading assignments and peer interactions during the discussion [7, p. 976].

H. N. McKnight was aimed to determine the impacts of Peer Instruction in a public college biology classroom. The research was a pretest-posttest, control group design. The study included 134 students registered in General Biology in the fall semester of 2014. The results showed that although there was an increase in achievement test scores in the treatment group compared to the control, the outcomes were not an essential difference between peer instructed class and traditional class [191, p. 110].

A. P. Fagen, C. H. Crouch, and E. Mazur tried to determine the opinions and thoughts of teachers using the Peer Teaching Method in their classrooms by conducting surveys. This study was conducted with 2750 participants from 34 countries. The results obtained from the data obtained from the surveys revealed that the method creates a positive ambience in the class environment, makes the lecture funny, increases the satisfaction of the students, and their participation in the lessons is high. In addition, the answers given to the questions stated that they did not care that the answer was wrong or absurd because it was formed by the common decisions of the students and they encouraged each other [207, p. 208].

Özcan (2017) investigated the effect of peer teaching method on teaching acids and bases in his doctoral thesis. In addition, within the scope of the study, the attitudes and opinions of the learners towards the peer teaching method and the attitudes of this method towards the chemistry lesson and discussion were also tried to be revealed. The study was conducted with 21 senior learners studying in a high school in Erzurum. The research was designed as an action research, and qualitative and quantitative data were collected. In the study, the lessons were taught with the peer teaching method for 5 weeks. As a result of the research, it was seen that the peer teaching method increased the academic success of the learners and increased the conceptual understanding of the students. In addition, it was stated that the learners' attitudes towards the applied method were mostly positive. It was determined that there was no statistically an important difference in learners' attitudes towards chemistry lesson and discussion. However, it was seen from the interviews

with the students that they expressed a positive opinion about the chemistry lesson and the discussion [262].

T. Gök compared peer teaching with traditional teaching in terms of students ' performance, ability, and self-confidence based on problem-solving. The semi-experimental research model was used in the study, which was attended by 98 high school 2nd-grade learners who participated in the physics course, and the application lasted 5 weeks. A statistically an important distinction was determined in favor of the peer instruction field treatment group based on the results of the physics success test and problem-solving confidence test applied to the students [17, p. 756].

D. Zingaro and L. Porter investigated the effects of traditional and peer teaching methods on learners' academic accomplishment and programming self-efficacy in Introduction to Computer Science 1 course for 12 weeks, including three 50-minute courses and one laboratory per week, with university students in the fall of 2012 in Canada. Throughout the research, while the lesson was taught with the peer teaching method in one of the two groups of learners, the other was taught with the traditional teaching method. Although the learners in the peer education group got higher mark in the final exam at the end of the study, there was no meaningful difference between the groups in terms of academic success. However, it was concluded that the peer teaching method significantly increases the learners' perceptions of programming self-efficacy. The researcher emphasized that with the peer teaching method, students are more interested in the lesson, they like the lessons more, and they gain self-efficacy against programming, besides getting high grades. He also stated that the success of the peer teaching method in increasing the perception of self-efficacy can be thought to be due to the fact that it offers many opportunities to students in order to receive fast and accurate feedback [230, p. 94].

P. Zhang, L. Ding, and E. Mazur investigated the peer teaching method's effect on university students' beliefs and attitudes towards the physics course introduction. The sample of the research consisted of 441 learners studying at Beijing Normal University in China. During the study, peer teaching method was used in three classes where these students were present, and the conventional teaching method was used in another classroom. In two of the groups in which the peer teaching method was used, the peer groups were constantly changed during the study process, and in the other group, the peer groups remained constant in this process. Research data were collected through an attitude questionnaire. According to the results of the research, there was no meaningful difference in the attitudes and beliefs of the learners in the classroom where traditional education was given. However, in the groups where the peer teaching method was used, it was observed that there was a positive increase in learners' attitudes and beliefs towards physics lesson. In addition, the change in the attitudes and beliefs of the students in the class with fixed peer groups was more positive compared to the learners in the classes with variable peer groups [263].

S. S. Tokgöz, in his doctoral dissertation, conducted a research on the academic accomplishment of 6th class primary school students in science lesson,

attitudes and remembering rates towards electric current. The study was carried out for three weeks with a total of 121 learners, 63 in the treatment group and 58 in the control group. The lectures were taught with the peer teaching method in the treatment group and with the conventional method in the control group. The Flowing Electricity Achievement Test and the scale of attitude towards Flowing Electricity were administered to all learners participating in the study as a pretest at the beginning of the study and as a posttest at the end of the study. As a result of the statistical analysis of the research data, it was determined that the peer teaching method had a meaningful positive influence on the academic accomplishment and retention rates of students. On the other hand, it was determined that there was no meaningful difference between the treatment and control groups in students' attitudes towards the course [20, p. 66].

H. Eryilmaz examined the effect of peer teaching method supported by concept tests on the academic accomplishment and attitudes of high school learners in physics course. The study was conducted with peer teaching method in the experimental group and traditional teaching method in the control group for three weeks. The sample of the study consisted of 92 treatment and 100 control group, totally 192 students. The Physics Attitude Test and the Physics Achievement Test were administered to both groups as a pretest and a posttest at the end of the instruction. As an outcome of the study, it was observed that the academic achievement of the students in the group where the lessons were taught with the peer teaching method increased significantly compared to the students in the group where the traditional teaching was done, but there was no important distinction between the groups in terms of attitude towards the course [16, p. 58].

P. J. Green has determined that by applying the Peer Teaching Method in astronomy lessons, better results can be achieved if the lessons are handled with care. In his study, Green determined that students' attendance, interest, and motivation were increased. In addition, he stated that it would be beneficial to apply the Peer Teaching Method in the lessons as it improves the scientific process abilities and communication skills of the learners, ensures that the concepts are learned correctly and effectively, and increases the satisfaction of the students [190, p. 46].

A. G. Şekercioğlu Çirkinoğlu examined the effect of peer teaching method on pre-service instructors' conceptual comprehension of electrostatics and their attitudes towards peer teaching method in her doctoral thesis. Within the scope of the research, the attitudes of instructor candidates towards physics lesson were also tried to be revealed. The sample of the study consisted of 157 students studying in different departments at Balıkesir University Necatibey Faculty of Education in the spring term of the 2007-2008 academic year. Pretest-posttest control group quasi-experimental research design was used in the study. Electrostatic Concept Test, Physics Attitude Questionnaire, Peer Teaching Attitude Questionnaire were used to collect quantitative data within the scope of the research, and also interviews were made with the participants. At the end of the research, it was revealed that the peer teaching method increased the conceptual understanding of the students. It has been

determined that the attitudes and opinions of the instructor candidates towards the applied method are positive. It was also stated that the attitudes of the participants towards the physics lesson were positive and there was no meaningful distinction between the groups. In addition to these, it was determined that instructor candidates have misconceptions about conductivity-non-conductivity, Coulomb force, electrical field, electrical potential and energy, Gauss's law and capacitance and situations that they have difficulties in understanding [166, p. 209].

R. Caceffo, G. Gama, and R. Azevedo compared the effect of three different teaching methods, Course Based Learning, Project Based Learning, and Peer Teaching Method, on learners' motivation in the introduction to computer science course. At the end of the study, it was concluded that learners and instructors have a positive perspective towards new technology and new teaching approaches. It was observed that the Peer teaching method and project-based learning method, which are active learning approaches, positively affected students' perceptions of learning and motivation. In the study, it was emphasized that in order for active learning methods to be implemented, trainers should spend more time in preparation for lessons than traditional lesson-based methods. Another point pointed out in the results of the research is that the classrooms where peer education is used in the evaluation process of the students are more unsuccessful than the classes in which project-based applications are performed. The reason for this was explained as the students' replies to the questions asked at the end of the lecture presentations in the classrooms where peer teaching was applied during the study using their smart phones and there was a dispersion in the classroom at this stage. For this reason, the researchers also stated that they were thinking of customizing the peer teaching method in the introduction to computer science course by developing a system that makes classroom preparation and evaluation partially autonomous in the future [199, p. 926].

A research was conducted to compare the peer teaching method used at Harvard University with the classroom communication systems used in MIT (Massachusetts Institute of Technology) and the broad group discussion. The findings of the research revealed that peer instruction technique is found to be more beneficial for students in terms of learning, it enables participants to participate more effectively in classroom discussions, and it is a more useful discussion method for teachers [21, p. 470].

S. P. Rao, and S. E. DiCarlo investigated the influence of peer teaching method on the success of learners in quizzes during the lesson in their examination with 256 first year students studying at Wayne State University medical school. In the study, which included 10 lesson hours of 50 minutes, the lessons were divided into 12-20 minutes segments, and the multiple-choice conceptual questions asked at the end of each of these sections were handled as quiz at the end of the episode. The questions in the quizzes are designed by dividing into three different levels. Level 1 questions are questions prepared to measure the permanence of the basic information processed in the relevant section. Level 2 questions are application and analysis questions prepared to measure how well students understand the relevant section.

Level 3 questions are used to measure synthesis and evaluation skills and require advanced mental performance. Throughout the study, a total of 35 questions were asked to the participants and their responses were recorded before and after the discussion. When the results were examined, it was determined that the rate of correct answers after the discussion was significantly higher in all three groups of questions than before the discussion (1st level questions BC = 94%, TS = 99%, 2nd level questions TE = 83%, TS = 99%, level 3 questions TE = 73%, TS = 99.8%) [204, p. 54].

M. C. James compared the usefulness of peer teaching in crowded and non-crowded classes. The findings of the study show that the lack of consensus among peers after the discussion was 7.6% in crowded grades and 36.8% in non-crowded grades [264].

T. Yıldırım and C. Canpolat aimed to investigate the influence of the peer instruction method learners' attitudes toward chemistry and on students' conceptive understanding for teaching about solutions at the high-school grade and to compare peer instruction with the traditional method. The study was carried out with 59 learners from 11th class in Artvin city in Turkey in the 2016-12017 education years. In the study, classes were randomly divided into the treatment group and the control group. The implementation step continued for four hours per week for five weeks. The data for the research was obtained using the concept test solutions and Attitude toward chemistry. The results indicated that the peer instruction method is more effective than the traditional method in support understanding of the concepts of chemical solutions. There were no statistically important distinction between the two groups on students' attitudes towards chemistry [19, p. 140].

In the project prepared by R. L. Miller, E. Santana-Vega, and M. S. Teller about the teaching of the General Mathematics course, the Peer Teaching Method used by Mazur in physics courses was taken as the basis [19, p. 195]. In this project, the questions called "Useful Questions" were used instead of Concept Tests. Prepared with the subsequent improvement of the concept test developed by Scott Pilzer, these questions are the kinds of questions that encourage discussion and suggestion, are open to interpretation, with more than one solution, perhaps with no solution at all. The project findings show that peer discussion makes it useful to use useful questions, as it allows you to reflect on your questions. In addition, final exam results explained that the method had an important influence on all students and all groups [195, p. 197].

N. Lasry, E. Mazur, and J. Watkins investigated the effect of peer teaching method on learners' academic accomplishment and dropout tendencies compared to conventional teaching method. The findings obtained in the study were compared with the results of the research conducted on the peer teaching method in Harvard University in 1991 and the relationship of this method with the academic level was tried to be revealed. The sample group of the study was divided into two as low and high academic readiness, and the difference in the effectiveness of the peer teaching

method between these groups was examined. The study was carried out with a total of 127 learners, 83 of them in the treatment group and 44 of them in the control group, within the scope of introduction to physics course in John Abbott College, where the education period is two years. At the end of the study, it was found that the academic accomplishment and conceptual comprehension of the group in which the peer teaching method was applied increased significantly among the groups with low academic readiness compared to the group in which the classical teaching method was applied. Similarly, it was found that the academic achievement and conceptual understanding of the group in which the peer teaching method was applied among the groups with a high level of academic readiness increased significantly compared to the group in which the classical teaching method was applied. In addition, at the end of the research, it was determined that 5% of the students in the class taught according to the peer teaching method did not take the final exam, but 25% of the learners who took the lesson according to the classical teaching method did not take the final exam. It was emphasized that this situation is parallel with the result that the rate of students not taking the final exam has continuously decreased during the peer teaching method used in the study conducted at Harvard University. As a result, this study has revealed that the peer teaching method produces positive results in all students, regardless of whether their academic level is low or not [181, p. 1067].

R. N. Cortright, H. L. Collins, and S. E. DiCarlo investigated the effect of peer teaching method on university learners' new problem-solving skills. The study was conducted with 38 learners who took the Physiology course. Before starting the application, the class was divided into two groups of 19, consisting of randomly selected students. In the study, first of all, the lecture was given theoretically. Then the same conceptual questions were asked to the groups on the subject. The groups alternately answered the questions first, one group individually and the other according to the peer teaching method. In other words, peer teaching method was applied alternately in both groups. As a result, it was observed that the rate of correct answers to conceptual questions increased when the peer teaching method was applied [201, p. 110].

C. Y. Chou and P. H. Lin, in their study conducted at Yuan Ze University in Taiwan, discussed in the discussion section, which is one of the basic elements of the peer teaching method, the students 'random selection of their friends to discuss, in terms of the efficiency of the discussions and the students' willingness to participate in the discussions. In addition, in order to evaluate the students' willingness to participate in the discussions in another way, at the stage where the groups were determined by the teacher, a grading system was used in which the correct answers given by the group members affected the scores of the other members to a certain percentage. According to this system, the student's score for each course was calculated as 40% of individual answers given before the discussion, 30% of individual answers given after the discussion, and 30% of the individual answers given by their friend after the discussion if they were two people in the discussion group. In cases where there are three people in the discussion groups, the answers

given by the members after the discussion affect the scores of other friends by 15%. The study, in which 84 students taking the computer programming course participated, was completed in 11 weeks in total. In the first 6 weeks, the discussion groups were determined differently for each lesson by the teacher and scored according to the grading system explained above. In the second stage, which lasted for 5 weeks, the students randomly sat down and just argued with their friends who were sitting next to them randomly. In addition, students were given points only for their own answers, and they did not get any points from the answers of their friends they discussed. In both applications, the students conveyed their answers and their level of confidence (I'm sure, not sure, just guessed) to the teacher using an electronic answering system. In addition, each student submitted a self-evaluation report to the teacher through the electronic answering system, indicating whether he participated in the discussions and whether this had an effect on his answer. As another data collection tool, at the end of the first 6-week phase, students were asked to write explanations for their options by applying a Likert-type scale consisting of 5 questions. When the results were analyzed, it was determined that when the discussion members were determined by the instructor, the percentage of learners participating in the discussion (80%) was significantly higher than the other application (60%). When the Likert scale results were examined, it was found that the vast majority of the students found it useful to discuss with their friends (95%), liked to argue with their friends (91%), that their friends' answers affected their scores (66%), but some students decreased their scores it was determined that they did not find this system fair (17%) [26, p. 844].

C. Turpen and N. D. Finkelstein, in their study at the University of Colorado, tried to identify the similarities and differences that emerged by observing the practices of 6 physics professors who teach their lessons using the peer teaching method in the same section. All of the classes in which the practices were carried out were crowded classes with 130 - 240 students and it was stated that the educational environments were similar. Data were collected using 3 different methods in the study that continued for a period. First, the researchers took observation notes by observing the classes of the practitioners in the first few weeks of the study, and based on these notes, they prepared an observation rubric. Comparing the data obtained from these rubrics with the practices of the professors, similar and different aspects are revealed and the characteristic features of these applications are quantified. As the second data collection method, semi-structured interviews were conducted with the participating professors at the end of the period when the application was completed, and questions were asked such as the differences of the lessons traditionally taught with their own practices, and in which situations a student can be considered as an active participant. Thirdly, the responses given by the participants to the concept questions were recorded with the electronic answer system and analyzed. When the results are examined, it was determined that three of the 6 professors who applied the peer teaching method almost never left the chair during the course and had a very limited interaction with the students. It was observed that the other three left the lectern and walked to the back rows among the students,

participated in the students' discussions and answered their questions. In addition, it was observed that there were differences in the time that the professors gave the students to answer the concept questions and to discuss afterwards - although they stated that they followed the same method in the interviews made with them - the methods of application differed from teacher to teacher. It has been emphasized that these differences may limit opportunities such as conceptual reasoning, discussion, and questioning that are aimed to be presented to students within the scope of peer education [265].

L. Porter, C. Bailey-Lee, and B. Simon investigated the difference between the academic accomplishment and dropout rates of learners using classical and peer teaching methods in four different computer science courses. In the research, the results were tried to be revealed based on the data obtained for 10 years. According to the results of the research, it was stated that the academic failure levels and dropout rates of the learners studying in the courses in which the peer teaching method was applied decreased by 61% per course compared to the students who studied in the courses where the classical teaching method was used. It has been emphasized that this rate is 20% in total in the classical teaching method, and decreases to 7% in the peer teaching method. Since the research lasts for 10 years, considering that the faculty members who teach the same course may change from year to year, the data of the students of the faculty members who teach the same course in one class according to the peer teaching method and in the other according to the traditional teaching method were analyzed separately. As a result of the study, the failure and withdrawal rates of students in classes with peer teaching method were still relatively low compared to others [177, p. 180].

M. F. Golde, C. L. Koeske, and R. McCreary investigated the effect of peer teaching method on university learners' academic accomplishment in the General Chemistry Laboratory-I course. The sample of the study consisted of a total of 148 participants, 39 of which were the treatment group and 109 were the control group. The theoretical lessons were taught with the peer teaching method in the experimental group and the classical method in the control group. At the end of the research, the achievement test consisting of open-ended questions was applied to both groups. For the answers given to the test questions, a scoring key was created such that "1 = poor, 2 = medium, 3 = good". In addition, for the clarity and length of the answers given to the test questions, a separate scoring system has been created such that "1 = 25% of the page is filled, 2 = 26-50% of the page is filled, 3 = 50% and above of the page". At the end of the study, the ratio of "3 = good" points in all scores of the group in which the lectures were taught with the peer teaching method (32%), the ratio of "3 = good" points in all points of the classical education group (18%) was determined. This situation was interpreted as the success percentage of the experimental group was significantly higher. Similar situation was observed as the ratio of the experimental group's "1 = poor" scores to all scores (34%), and the control group's "1 = poor" scores to all scores (50%). In addition, when evaluated in the context of the length and clarity of the replies, it was observed that the percentage

of participants in the treatment group to fill half and above the page (Treatment Group = 28%, Control Group = 12%) was noticeably higher [196, p. 804].

Y. Z. Olpak, F. G. Yilmaz, and R. Yilmaz, created a form with 179 pre-school teacher candidates to measure the attitudes of students towards peer education. The peer instruction evaluation form has 25 items. According to the results of the research carried out, the participants thought that the peer instruction method is obvious and chasing is easy. Furthermore, most of the students described peer teaching as interesting and entertaining. Additionally, participants stated that the peer education method helped to better understand the lesson subjects and to go beyond their previous knowledge levels. Finally, it is seen that the participants, in the same opinion as the previous studies, also stated that using the peer teaching method increased their confidence, participation and motivation [266].

1.3.1 The important studies about peer instruction

	Purpose	samples	result
Lacaba, Magalona, and G. Lacaba 2018	To explore the effect of peer instruction approach on learners' accomplishment in a mathematics lesson.	Third-grade mathematics students in the Philippines	The outcomes of the research determined that the implementation of the Peer instruction approach has a positive impact on increasing the academic accomplishment in mathematics of Grade 3 pupils.
Demirel (2013)	To examine the effect of the use of peer instruction in mathematics lesson on student's attitude, success and knowledge permanence.	41 students attending 6th grade of primary education in Kayseri	The course success of the group in which peer instruction is used is improved compared to the group in which traditional education is used and the information persistence is higher, but it has no effect on attitude towards the course.
Cronhjort, Filipsson, and Weurlander (2013)	To compare the effect of peer instruction instead of the conventional method on students learning and student perceptions learning in a Calculus course.	Approximately 2 000 beginning engineering students in University of KTH Royal Institute of	The results showed that peer instruction is an efficient teaching and learning method that serves students to improve deeper conceptual comprehension.

		Technology in Stockholm, Sweden	
Oloo, Mutsotso and Masibo (2016)	To observe the influence of peer teaching during the education process on the learners' performance in mathematics.	167 mathematics students in twelve randomly selected primary schools in Bungoma	Peer teaching approach increases students' achievement in mathematics, students' motivation to learn mathematics, and improves comprehension of mathematical notions and establishes confidence in the learners.
Olpak, Baltaci, and Arican (2018)	To investigate the influence of the peer education process, on secondary school mathematics educators' success in the topic statistics and probability.	The second period of the 2016-2017 academic year with 46 third course participants in statistics and probability	Peer instruction increasing students' success and providing learning activities.
Ouko, Aurah, and Amadalo (2015)	To examine the effect of peer instruction approach on learners' success in vectors.	The research was managed in Kenya along the Kenya-Uganda Border with 479 students.	The results showed that the peer instruction method increased students' success in vector lessons more than the conventional teaching method.
Awinoouko (2018)	To determine the Role of Peer Teaching in Problem Solving Skills of Students' Problem Perceptions.	The study was conducted in Bungoma. Proportional Sampling was used to	The results show that most of the students perceive themselves to solve problems in mathematics after peer teaching.

		select 300 participants.	
Uesaka and Manalo (2007)	To investigate the effectiveness of peer teaching to solve math word problems using diagrams would.	57 8th-class learners from public secondary schools in Tokyo.	The original finding was that in post-teaching evaluation, those in experimental conditions had to prove that diagrams were spontaneously used more in their attempts to solve the given math word problems.
Pilzer (2001)	To investigate the effectiveness of peer instruction in the reasoning skill and retention.	Albright College students in the USA	After checking the responses of the open-ended questions about the technique used in this research, peer discussion was seen to improve the learning process.
Ferreira, Nicola, and Figueiredo (2011)	To determine the impact of Peer Instruction method in an introductory Calculus course of an Engineering	558 Engineering students in a calculus course in Porto	Peer Instruction was therefore successful in getting low-level students to fully participate in the course and created a good atmosphere
Akay (2011)	To measure the influence of the peer education process on learners 'mathematics success and learners' attitudes to the mathematics lesson.	112 eighth grade students studying in Istanbul city	It was observed that the learners in the treatment group used in peer instruction were higher in terms of math achievement and attitude towards mathematics than the groups used in the traditional method.
Campit and Garin (2017)	To determine the influence of Peer Instruction technique on the attitudes towards mathematics.	30 second-course college students at the State University of Pangasinan.	A meaningful alteration in the attitudes of the learners in the peer learning group was obtained
Abdelkari	To examine bachelor	32 bachelor	The results indicate that

m, Abuiyada, and Siddiui (2016)	students' attitudes towards mathematics after peer instruction method.	students in "Mathematics for Social Sciences 1" in Oman	the learners after the peer instruction had a more positive image of mathematics, which makes strong the influence of peer instruction on the attitude.
Allison (2012)	Investigating the impact of peer instruction on student success and motivation.	168 8th grade students attending math class in Georgia, USA.	The success of the peer instruction class was found to be statistically more successful than the non-peer instruction class, and a partial differ ence in student motivation was found to be in favor of the group receiving peer instruction.
Abdelkarim and Abuiyada (2016)	To investigate the effects of peer teaching on mathematics academic success of bachelor students.	32 bachelor students in "Mathematics for Social Sciences 1" in Oman	The findings suggest that peer teaching method improves students ' success
Yavuz (2014)	To examine the effect of peer instruction on students' success and attitudes of 7th Grade students on the topic of Rational Numbers.	472 students attending secondary school in Ankara province	Lesson groups taught with peer instruction are more successful than traditional method groups, but there is no significant difference in attitude towards rational numbers.
Lucas (2009)	To show that peer instruction and i-clickers improve learner attendance and comprehension.	The study included 81 learners of the Don Mariano Marcos Memorial State University	The results of the study indicated that peer instruction and i-clickers enhance learner participation and understanding.
Crouch	To determine the influence	In 1990-	Development of students '

and Mazur (2001)	of peer teaching during the physics lecture.	2000, a different number of undergraduate students attended basic physics courses at Harvard University each year.	ability to solve mathematical problems and their conceptual understanding
McKnight (2015)	To determine the impacts of Peer Instruction in a public college biology classroom.	The study included 134 students registered in General Biology in the fall semester of 2014.	The results showed that although there was an increase in test scores in the control group compared to the experimental, the results were not an important difference between peer instructed class and traditional class.
Gök (2013)	To compare peer teaching with traditional teaching in terms of students ' performance, ability and self-confidence based on problem-solving.	98 high school 2nd-class learners who participated in the physics course,	A statistically important difference was determined in favor of the peer instruction field experimental group based on the results of the physics success Test and Problem-solving confidence test applied to the students.
Fagen, Crouch and Mazur (2002)	To determine the opinions and thoughts of teachers using the Peer Teaching Method in their class environment by conducting surveys.	This study was conducted with 2750 participants from 34 countries	The results showed that the method creates a positive atmosphere in the class environment, makes the lesson enjoyable, increases the satisfaction of the students, and their participation in the lessons is high
Zhang, Ding, and	To examine the effect of Peer Teaching Method on	441 students at Beijing	They found that there was no change in attitudes and

Mazur (2017)	learners' attitudes and beliefs towards basic physics.	Normal University	beliefs in the classes where conventional education was provided, but there was an improvement in students' attitudes and beliefs towards physics in the classes where peer education was provided.
Rao ve Di Carlo (2000)	Increase students ' participation in class with peer instruction.	1.256 students of first-course medical physiology	It has shown that peer instruction increases students 'level of comprehension and also improves students' ability to develop and synthesize information.
Nicol ve Boyle (2003)	Comparing the class-wide dialogue discussion methods with the discussions in peer instruction.	117 university students studying Mechanical Engineering in the UK	Peer discussions were found to be more effective than class-wide discussions, and also class-wide discussions sometimes extended and reduced interest in the class.
Miller, Santana-Vega, and Teller (2006)	To research on the effects of peer instruction technique on mathematics learners.	General Mathematics course students.	The project findings show that peer discussion makes it useful to use useful questions, as it allows you to reflect on your questions. In addition, final exam outcomes showed that the technique had an important effect on all students and all groups.
Green (2003)	To determine the effect of Peer Teaching Method in astronomy lessons.	Astronomy course students	Green determined that students' attendance, interest, and motivation were increased.

Yıldırım and Canpolat (2019)	To determine the influence of the peer instruction method learners' attitudes toward chemistry and on students' conceptive understanding for teaching about solutions at the high-school grade and to compare peer instruction with the traditional method.	59 students from 11th grade in Artvin city in Turkey in the 2016-12017 education years.	The results indicated that the peer instruction method is more effective than the conventional method in support understanding of the concepts of chemical solutions and no statistically important distinctions between the two groups on students' attitudes towards chemistry.
Eryilmaz (2004)	To examine the effect of peer teaching method supported by concept tests on the academic accomplishment and attitudes of high school learners in physics course.	192 high school students.	As a result of the study, it was observed that the academic achievement of the participants in the group where the lessons were taught with the peer teaching method increased significantly compared to the students in the group where the conventional teaching was done, but there was no meaningful distinction between the groups in terms of attitude towards the course.
Sencar Tokgöz (2007)	To determine effect of peer instruction on the academic in science lesson, attitudes and remembering rates towards electric current.	121 6th grade primary school students	As a result of the statistical analysis of the research data, it was determined that the peer teaching method had an important positive effect on the academic accomplishment and retention rates of students. On the other hand, it was determined that there was no important difference between the treatment and control groups in students'

			attitudes towards the course.
Cortright, Collins, and DiCarlo (2005)	To investigate the impact of peer teaching method on university learners' new problem-solving abilities.	38 Physiology course students	As a result, it was observed that the rate of correct answers to conceptual questions increased when the peer teaching method was applied.
Lasry et al. (2008)	To investigate the influence of peer teaching method on learners' academic accomplishment and dropout tendencies compared to traditional teaching method.	The study was carried out with a total of 127 students, within the scope of introduction to physics course in John Abbott College, where the education period is two years.	As a result, this study has revealed that the peer teaching method produces positive results in all students, regardless of whether their academic level is low or not.
Olpak, Yilmaz, and Yilmaz (2017)	To create the peer instruction evaluation form.	with 179 pre-school teacher candidates to measure the attitudes of students towards peer education	The most of the students described peer teaching as interesting and entertaining. Additionally, participants stated that the peer education method helped to better understand the lesson subjects and to go beyond their previous knowledge levels. It is seen that the participants, in the same

			opinion as the previous studies, also stated that using the peer instruction increased their confidence, participation and motivation.
Porter, Bailey-Lee, and Simon (2013)	To investigate the difference between the academic accomplishment and dropout rates of learners using classical and peer teaching methods in four different computer science courses	10 years of instruction of 4 separate courses spanning 16 peer instruction approach course instances	As a result of the study, the failure and withdrawal rates of students in classes with peer teaching method were still relatively low compared to others.

1.3.2 Summary of Literature Review

In Summary, the benefits of mathematics in the educational life in secondary education and college-level cannot be denied. In conclusion of the first part, it can be observed that the traditional method is not enough to improve the students' mathematics achievement and their attitudes towards the mathematics lessons. Additionally, it is also not enough to increase the psychological and pedagogical development of the learners. It has been concluded that active teaching methods have a greater effect on students' achievement and attitudes than the traditional teaching method. Peer Instruction is an active learning and cooperative learning method. When we examine the researches, we see that a lot of study has been done in the field of peer teaching abroad. The number of studies in our country is not sufficient this is because the importance of peer education is unknown. Also, there is not enough study in mathematics in our country. Our aim in this research study is to examine the impacts of peer instruction on mathematics lesson. This research, which aims to present experimental results to students and mathematics teachers about how feasible education is applicable in mathematics teaching and how it will affect teaching processes, is important in terms of introducing different approaches in mathematics education. There are many positive benefits to the use of Peer Instruction in classroom teaching. Current literature indicates that Current literature indicates that peer instruction has an impact on academic achievement (C. H. Crouch and E. Mazur [7, p. 975]; F. Demirel [15, p. 88]; Eryilmaz [16, p. 59]; T. Gök [17, p. 757]; G. Akay [23, p. 90]; H. N. McKnight [191, p. 110]; T. Yıldırım and N. Canpolat [198, p.

78]; S. P. Rao and S. E. DiCarlo [204, p. 54]; A. P. Fagen, C. H. Crouch, and E. Mazur [207, p. 209]; R. E. Abdelkarim & E. Abuiyada [244, p. 130]; A. B. Lacaba, J. D. Magalona & T. V. G. Lacaba [245, p. 9]; Oloo, S. N. Mutsotso, & E. N. Masibo [246, p. 14]; Y. Z. Olpak, S. Baltaci, & M. Arican [247, p. 2328]; S. Ouko, C. Aurah, & M. Amadalo [248, p. 179]), attitudes towards lesson (J. B. Campit & R. M. Garin's [14, p. 14]; F. Demirel [15, p. 89]; G. Akay [23, p. 94]; R. Abdelkarim, R. Abuiyada, & S. A. Siddiui [258, p. 1515]; O. C. Yavuz [259, p.81]; P. Zhang, L. Ding, and E. Mazur [263, p. 7]), motivation (T. H. Allison [13, p. 99]; P. J. Green [190, p. 78]; Oloo, S. N. Mutsotso, & E. N. Masibo [246, p. 15]), problem-solving ability (T. H. Allison [13, p. 99]; T. Gök [17, p. 758]; Y. Uesaka and E. Manalo [261, p. 681), attendance, self-confidence (F. Demirel [15, p. 89]; Oloo, S. N. Mutsotso, & E. N. Masibo [246, p. 15]), retention (F. Demirel [15, p. 90]).

2. EXPERIMENTAL WORK OF IMPLEMENTING THE PEER INSTRUCTION

The method of teaching math is one of the meaningful factors affecting accomplishment in mathematics. Because how a person learns mathematics is closely related to one's perspective of mathematics. In the studies conducted, it is seen that the teaching strategies and methods applied in mathematics courses are quite effective. Today, it is accepted that some of the problems in teaching mathematics stemmed from the teaching strategies and techniques applied in the classroom [267]. Learners in an active learning environment are much more capable of reaching success [203, p. 163]. Classrooms that use Peer Instruction could be useful for schools intent on accomplishing this target. This chapter has presented the technique used in the study and provided more information on it, participants and selected sample, data collection instruments, data collection procedure, analyses of data, results.

2.1 Research Design

The main idea behind the research study is to examine the influence of peer instruction technique on the 9th-level learners' mathematics academic accomplishment and attitudes towards mathematics in the topic Trigonometry. Besides, in this research, we sought to determine any gender differences in mathematics accomplishment and mathematics attitude. Similar to J. R. Fraenkel, and N. E. Wallen study, in the present research, the static-group pretest-posttest technique was incorporated and implemented. The design of the research is illustrated from the table below [268].

Table 1 - The research design

	Experimental	Control Group

	Group	
Pretest	+	+
Implementation	+	
Posttest	+	+
evaluation form	+	

As stated in Table 1, in this study the participants were from two groups, one of the two groups as the experimental group and the other of them as the control group. Data were collected from both groups at the same time and twice; the initial one was the pretest (before the application) and the last was taken the posttest (after the application).

Participants

The sample taken was a group of ninth-grade students from three different school locations in one city, Almaty in Kazakhstan. 122 students were male, and 49 students were female. The student population was multicultural.

The school uses an educational system called gymnasium and offers most of the classes such as mathematics, physics, biology, chemistry, and computer in English except Russian literature and a few Kazakh lessons. Students are mostly from middle and upper socioeconomic status. Students are admitted from the 7th grade and the school lasts for five years. There are weekly six hours of mathematics, in the 9th, 10th, and 11th classes. Graduates from these schools usually prefer universities outside Kazakhstan such as China, Korea, the USA, the UK, Turkey, and Singapore. Since this is a state school, the state meets all expenses of the students except food. Parents are only charged to pay for lunch. Before graduating, all students take a national test. Last year (2018) 97.4% of the students were successful in the national exam and got the right to enroll in universities.

The classes were chosen for convenience because the researcher had worked as a mathematics teacher in this school in the previous years. The students are fluent in Kazakh and Russian, and they also speak English and Turkish at the upper intermediate level. In the beginning, they were told about the aim and scope of the study being conducted including a short explanation of the expectations and predictions of the students regarding the curriculum to be covered. The students were all volunteers for the study.

Table 2 - Distribution of Students in Experimental and Control Groups by Gender

Number of Male and Female Students			
Groups	Male	Female	Total
Experimental group	46	23	69
Control group	76	26	102

Instruments

The research aimed to investigate the impact of peer education method as well as its influence on students' gender on the 9th class students' mathematics accomplishment and attitudes towards trigonometry topic instruction. As a quantitative data collection tool in this study; The "Mathematics Achievement Test" (Appendix 1) was used to evaluate learners' understanding of the topics in mathematics, the "Mathematics Attitude Scale" (Appendix 2) to examine their attitudes towards mathematics. "The Peer Instruction Student Evaluation Form" was used as a qualitative data collection tool.

Mathematics Achievement Test (MAT)

A mathematics achievement test is an assessment tool used to measure student performance. R. A. Morales argues that when making an important valuation of an assessment tool, two things have to be carefully considered; the reliability of the tool and the validity [269]. Our mathematics achievement test was administered to 68 10th-grade students as a pilot study and comprised of 39 items. The items were systematically developed to test student knowledge and comprehension in the topic of Algebra. After data collection, Item difficulty, KR20, Item discrimination and Point biserial correlation was done on the data. It was checked by two experts after being developed who gave their own thoughts and suggestions regarding it.

Table 3 - Mathematics Achievement Test Pilot Application Analysis Results

Item	**p**	**D**	**pbc**	**KR20**	**Item**	**p**	**D**	**pbc**	**KR20**
1	**0.14**	**-0.03**	**-0.01**	**.854**	21	0.53	0.50	0.90	.847
2	**0.94**	**0.06**	**0.19**	**.850**	22	0.30	0.26	0.60	.844
3	**0.94**	**0.06**	**0.17**	**.850**	23	0.42	0.44	0.86	.836
4	**0.94**	**0.06**	**0.23**	**.851**	24	0.50	0.29	0.54	.845
5	0.30	**0.09**	**0.25**	**.851**	25	0.42	0.38	0.66	.838
6	0.78	**0.12**	**0.23**	**.851**	26	0.42	0.38	0.80	.844
7	0.61	0.41	0.80	.840	27	0.36	0.32	0.72	.841
8	0.39	0.18	0.42	**.851**	28	0.75	0.20	0.53	.839
9	0.78	0.12	0.28	**.851**	29	0.44	0.35	0.75	.841
10	0.69	0.32	0.70	.844	30	0.47	0.32	0.62	.846
11	0.53	0.32	0.58	.846	**31**	0.25	**-0.09**	**-0.17**	.841
12	0.30	0.26	0.58	.845	32	0.33	0.18	0.39	.843
13	**0.00**	**0.00**	**0.00**	**.852**	33	0.75	0.26	0.64	**.859**
14	0.47	0.44	0.83	.840	34	0.53	0.38	0.75	.849
15	0.69	0.27	0.56	.845	35	0.44	0.41	0.78	.844

16	0.75	0.20	0.47	.847	**36**	**0.22**	**0.24**	0.64	.841
17	0.36	**0.09**	**0.06**	**.854**	**37**	**0.06**	**0.06**	0.39	.842
18	**0.06**	**0.06**	**0.19**	.850	**38**	**0.06**	**0.06**	0.39	.843
19	0.42	0.26	0.53	**.856**	**39**	0.42	**0.03**	**0.06**	.848
20	0.50	0.29	0.64	.850					

Note: p: Item difficulty, D: Discrimination index, pbc: Point bi-serial correlation, KR20: KR20 if item deleted

As analysis was being done, the items were being grouped as good and acceptable or improper as per the standards of the test. According to K. Quaigrain and A. K. Arhin in Item difficulty, the standard items considered good and acceptable range between 0.2 and 0.9 while in Item discrimination, the standard items considered good and acceptable are those >0.19 [270]. When it comes to point biserial correlation, the items are grouped as either good or very good. The items ranging between 0.2 and 0.39 are considered good while those ranging between 0.4 and 0.7 are considered very good. Therefore, if an item had inconsistencies in two or more of the analytical statistical groupings, they were removed and grouped as improper and unacceptable. In our case, 14 of the 39 items were done away with because of inconsistencies after finding they had improper values with two or more of the statistical groups.

According to l. M. Rudner and W. D. Schafer, Kuder-Richardson Formula 20 (KR20) was required to conduct an internal consistency check which focuses on the extent to which the items are correlated with each other [271]. They report that for a more reliable test, the coefficients of the KR20 statistic should range between 0.8 and 0.9, which indicates a high reliability although a test with coefficients ranging between 0.5 or 0.6 may also suffice. In this study, our coefficient was initially found to be 0.850 but after eliminating the unnecessary items, it was re-calculated and determined to be 0.877. In the end, because of the high reliability, validity and consistency of the data, the final 25 items were used in the main study.

Table 4 - Distribution of items according to topics and bloom's taxonomy level

Trigonometry	Remembering	Understanding	Application	Analysis	Evaluating	Creating
Fundamentals of Trigonometry	1	3				
Right Triangle Trigonometry			3	1		
Trigonometry Functions of Real Numbers	1	1	1	3		
Trigonometric Theorems and		1	7	3		

Formulas						

The Attitude towards Mathematics Scale (ATMS)

In this research, a 5-point Likert-type math attitude scale developed by P. Aşkar was used to determine the attitudes of students related to the mathematics lesson [272]. This scale, which will determine the attitude towards mathematics lesson; It contains 20 items consisting of 10 positive and 10 negative statements. These 20 items are scaled in five categories as "I Strongly Disagree", "I Disagree", "Neutral", "I Agree" and "I Strongly Agree" (Appendix 2). The reliability coefficient of the attitude scale towards mathematics course was calculated by P. Aşkar with Cronbach Alpha and found to be 0.96. As the Cronbach Alpha Reliability Coefficient was high as a result of the application of the Mathematics Attitude Scale by other researchers, the pilot application of the scale was not required in this study and was applied to the study group. Before using the mathematics attitude scale in the research, the researcher Dr. Petek Aşkar submitted his request to use the scale and after the positive response, this scale was taken as a data collection tool. During the application process, students were given explanations about the scale and given the necessary time to answer the questions.

The Peer Instruction student evaluation form

The Peer Instruction Student Evaluation Form developed by Y. Z. Olpak, F. G. Yilmaz, and R. Yilmaz consisted of 25 Likert-type items in three sub-sections: Learner assessments concerning the Peer Instruction method, student assessments regarding the multiple choice conceptual questions, and learner assessments regarding the peer discussions. Five choices were available for each item. The learners were expected to select only one option which represents their idea from the choices given; (1) strongly disagree, (2) disagree, (3) Neutral, (4) agree, and (5) strongly agree. Possible scores on the Peer Instruction Student Evaluation form scale range from 90. to 95. For internal consistency, Cronbach's alpha for the Peer Instruction Student Evaluation form was measured as .92 and is regarded as high in social sciences as stated by Fraenkel and Wallen [266].

Data Collection Procedure

The role of this current study is to show the influence of peer instruction technique as well as on gender using the 9th-class learners' mathematics accomplishment in trigonometry course and attitudes towards mathematics. As a next step of the pilot study, as represented above, regarding the participants' responses and comprehension of the lessons, there were some changes or removal of the items in the test. The program and plan for the 2019-2020 academic year mathematics course have been developed and prepared. There were three different secondary schools, and two groups (experimental and control) had 171 learners who were favourable for this research. At these schools, before the application, the purpose of the research and the

processes were described to the participants. Since the researcher was an instructor in both groups, he gave the participants brief information about the processes. After the students were informed about the work to be done in the first lesson, the mathematics accomplishment test, and attitude scale were implemented. The students were asked to complete the test within 60 minutes, and then the procedure was started. The application continued for 40 class hours in 10 weeks, and each lesson period was 40 minutes. A similar accomplishment test and the attitude survey were applied to both groups as a posttest following the treatment period. A period of 10 weeks was given between the pretest and the posttest; during this time, the application was made, and after that, all students in both groups completed the tests individually. Subsequently, the peer instruction student assessment form was applied in the treatment group to measure whether the peer instruction method influenced students' attitudes towards peer instruction.

2.2 Lesson Design

Teaching in Experimental and Control Groups

During the 10-week education provided, the peer education method was used for the students in the treatment group, unlike those in the control group. During the training, all procedures and activities were the same in both groups, except for the peer education method. Before the application, a treatment group and a control group were selected with an unbiased selection for the study. In the lead of the information received from school administrators and mathematics teachers in determining the classes to be used in the study, two equivalent branches were determined according to their success grades. The experimental group was informed about the process of the peer instruction method and that they would study in groups of randomly two or three. However, the control group participants were expected to work on their work individually each as in the traditional teaching; they were supposed to direct their questions to the teacher only not their classmates. Also, the lectures were conducted on a large screen in both classes so that the participants could easily follow. The questions were projected onto the board using an overhead projector. Students were directed to think with questions. Details of the procedure in the treatment and control groups are given below.

Table 5 - Mathematics subjects in Experimental and Control Groups

Week	Hour in a week	Hour	Subject	Experimental group	Control group
1	1	1	Pretest	+	+
	1	1	Angles	+	+
	1	1	Angles and the Unit Circle	+	+

2	1	1	Coterminal Angles	+	+
	1	1	Trigonometric Ratios in Right Triangles	+	+
	1	1	Special Triangles and Ratios	+	+
3	1	1	Special Triangles and Ratios	+	+
	1	1	Basic trigonometric Identities	+	+
	1	1	Basic trigonometric Identities	+	+
4	1	1	Basic trigonometric Identities	+	+
	1	1	Trigonometric Function in Unit Circle	+	+
	1	1	cos and sin functions, properties	+	+

5 - continuation of the table

5	1	1	tan and cot functions, properties	+	+
	1	1	Reduction Formulas	+	+
	1	1	Reduction Formulas	+	+
6	1	1	Reduction Formulas	+	+
	1	1	Finding Missing Ratios When a Ratio is Given	+	+
	1	1	Sum and Difference Formulas	+	+
7	1	1	Sum and Difference Formulas	+	+
	1	1	Sum and Difference Formulas	+	+
	1	1	Half Angle Formulas	+	+
8	1	1	Half Angle Formulas	+	+
	1	1	Half Angle Formulas	+	+
	1	1	Sum to Product Formulas	+	+

9	1	1	Sum to Product Formulas	+	+
	1	1	Sum to Product Formulas	+	+
	1	1	Product to Sum Formulas	+	+
10	1	1	Product to Sum Formulas	+	+
	1	1	Product to Sum Formulas	+	+
	1	1	Posttest	+	+

Implementation in the Control Group

Mathematics Achievement Test and Mathematics Attitude Scale were applied to the students in the control group as a pretest. The "Trigonometry" unit in the control group consisting of 102 students was processed in 40 lesson hours with the traditional teaching method. The teacher is active in the traditional teaching method. More straight narration and question-answer techniques were used. The teacher started each lesson by doing a short repetition of what was learned in the previous lesson and presented the lesson using the appropriate materials and techniques. At the end of each lesson, the questions were asked to the students and the shortcomings were eliminated and the summary of the lesson was made. At the end of the lecture, Mathematics Achievement Test and Mathematics Attitude Scale were applied as posttest.

Implementation in Experimental Group

The work in the treatment group was carried out by three mathematics teachers at three different schools. Mathematics Achievement Test and Math Attitude Scale were applied to the students in the treatment group as pretests. In the experimental group consisting of 69 students, the trigonometry unit was processed in 40 lesson hours by the peer instruction method. The participants in the class to be peer educated were randomly matched with the help of the mathematics teacher who entered their classes. Participants were informed about peer instruction method by the teachers before the lesson. During the briefing, information was given about peer teaching practices, conceptual tests and peer discussion. The teachers made observations by traveling among the students at the experimental stage and intervened when there was a lack of practice. The Mathematics Achievement Test and Mathematics Attitude Scale, which was used as a pretest at the beginning of the application, were also applied to the students as a posttest, and the influence of the instruction was examined by comparing the results with the pretest results.

Sample Lesson 1

The 40-minute lesson began with greetings and I asked the students to show me the previous assignment that I had sent to them. After a 5-minute review and check, we continued with the lesson of the day. First and foremost, I explained the new topic of Trigonometry Identities to the students in class in a short 7-minute lecture. We reviewed all the 3 proofs of the topic: Pythagorean identities, Tangent and cotangent identities, and Reciprocal identities.

Proof 1: Since sin θ = opposite/hypotenuse and cos θ = adjacent/hypotenuse therefore, $sin^2\theta + cos^2\theta = 1$

Proof 2: Since tan θ= opposite/adjacent and sec θ = adjacent/ hypotenuse therefore, $tan^2\theta + 1 = sec^2\theta$

Proof 3: Since cot θ = adjacent/ hypotenuse and csc θ = hypotenuse/ opposite, therefore, $cot^2\theta + 1 = csc^2\theta$

After the short lecture, we moved on to attempting conceptual questions with the implementation of Peer Instruction to test student understanding through the number of correct answers. The total number of students in class was 24.

1. *Evaluate: (1 - $cos^2\theta$) $csc^2\theta$ =?*
   A. 2 B. 1 C. 0 D. 5

In the first question, I displayed the question on the board and asked the students to attempt it. After 3 minutes of handling the question and 2 minutes of checking the correct answers, the correct answers were more than 70% and hence there was no need to move to the next stage of peer discussion. I explained the question. The question was relatively simple and easy to do.

Distribution of first responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	2	18	3	1

Explanation: Let A = (1 - $\cos^2\theta$) $\csc^2\theta$

A = (1 - $\cos^2\theta$) $\csc^2\theta$

Because $\sin^2\theta + \cos^2\theta = 1$, we have $\sin^2\theta = 1 - \cos^2\theta$

Then, A = $\sin^2\theta \cdot \csc^2\theta$

A = $\sin^2\theta \cdot (1/\sin^2\theta)$

$A = \sin^2\theta / \sin^2\theta$

$A = 1$

2. *Evaluate:*
   tan θ sin θ + cos θ =?

   A. sin θ B. cos θ C. sec θ D. tan θ

In the second question, I gave the students 3 minutes to attempt the question and after checking the answers, only 50% of the students had correct answers. We moved on to the next step of peer discussion for an additional 4 minutes. The number of correct answers after checking rose to 90% after the peer discussion. Again at the end of the application of peer instruction I explained the 2nd Question.

Distribution of first responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	1	8	12	3

Distribution of second responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	0	1	22	1

Explanation: Let A = tan θ sin θ + cos θ and B = sec θ.

A tan θ sin θ + cos θ, therefore after expanding, it becomes, A = (sin θ/cos θ) · sin θ + cos

$A = (\sin^2\theta/\cos\theta) + \cos\theta$

$A = (\sin^2\theta/\cos\theta) + (\cos^2\theta/\cos\theta)$

$A = (\sin^2\theta + \cos^2\theta) / \cos\theta$

$A = 1 / \cos\theta$

$A = \sec\theta$

3. *Solve: cos θ/ (1 - tan θ) + sin θ/ (1 - cot θ) =?*

A. cos θ B. sin θ C. cos θ + sin θ D. sin θ + tan θ

We moved on to the third question in the concept questions prepared. I gave the students 3 minutes to attempt doing the question and I checked the number of correct answers, which was 60% of the class. We then moved on to the peer discussion for an additional 4 minutes and the number of correct answers again rose to 90% of the class. As a last step of peer instruction I explained the 3rd question.

Distribution of first responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	2	2	14	6

Distribution of second responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	0	0	22	2

Explanation: Let A = cos θ/(1 - tan θ) + sin θ/(1 - cot θ) and

B = sin θ + cos θ

A = cos θ/ {1 - (sin θ/cos θ)} + sin θ/ {1 - (cos θ/sin θ)}

A = $\cos^2\theta$/ (cos θ - sin θ) + $\sin^2\theta$/ (sin θ - cos θ) therefore,

A = $\cos^2\theta$/(cos θ - sin θ) - $\sin^2\theta$/(cos θ - sin θ)

A = ($\cos^2\theta$ - $\sin^2\theta$) / (cos θ - sin θ)

A = [(cos θ + sin θ) (cos θ - sin θ)] / (cos θ - sin θ) and finally,

A = (cos θ + sin θ)

4. *Evaluate: (tan θ + sec θ - 1)/ (tan θ - sec θ + 1) =?*

A. (1 + sin θ)/cos θ B. (1 + cos θ)/sin θ C. (1 + tan θ)/cos θ D. 1 + sin θ

In the last question, which was relatively harder, I gave the students 4 minutes to attempt doing the question. Afterwards, the number of correct answers in class was just 10% after checking. We then moved on to the peer discussion for an additional 4 minutes and the number of correct answers by the students rose to 85% of the class. The lesson then ended after the assigned 40 minutes and the data was collected. For the last step of implementation of peer instruction I explained the last question for this *lesson.*

Distribution of first responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	3	10	8	3

Distribution of second responses

CHOICE	A	B	C	D
NUMBER OF STUDENTS	20	3	1	0

Explanation: Let $A = (\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1)$ and

$B = (1 + \sin\theta)/\cos\theta$.

$A = (\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1)$

$A = [(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)]/ (\tan\theta - \sec\theta + 1)$

$A = \{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)\}/ (\tan\theta - \sec\theta + 1)$ therefore,

$A = \{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta + 1)\}/ (\tan\theta - \sec\theta + 1)$

$A = \tan\theta + \sec\theta$

$A = (\sin\theta/\cos\theta) + (1/\cos\theta)$

$A = (\sin\theta + 1)/\cos\theta$ finally,

$A = (1 + \sin\theta)/\cos\theta$

Sample Lesson 2

Proofs

Proof 1: Cosine to Sine

Step 1: In deriving the first cofunction identity, we use the difference formula or the subtraction formula for cosine; we have cos (π/2 – u) = cos (π/2) cos (u) + sin (π/2) sin (u)

Step 2: Evaluate the trigonometric functions that are solvable. cos (π/2 – u) = (0) cos (u) + (1) sin (u)

Step 3: Simplify the expression. As a result, this gives us formula (1) cos (π/2 – u) = sin (u)

Proof 2: Sine to Cosine

Step 1: We can use the result in proof 1 to prove the second cofunction identity. If we substitute π/2 – v in the first formula, we obtain cos [π/2 – (π/2 – v)] = sin (π/2 – v)

Step 2: Evaluate the value of trigonometric functions that are solvable. cos (v) = sin (π/2 – v)

Step 3: Since the symbol v is arbitrary, the derived equation is equivalent to the second cofunction formula. cos (u) = sin (π/2 – u)

Proof 3: Tangent to Cotangent

Step 1: Using the tangent identity, cofunction formulas 1 and 2, and the cotangent identity, we obtain proof for the third formula: tan (π/2 – u) = [sin (π/2 – u)] / [cos (π/2 – u)]

Step 2: Simplify the trigonometric expression. tan (π/2 – u) = cos (u) / sin (u) tan (π/2 – u) = cot (u)

Method

1. After the students' correct answers exceeded 80% on the first question, we moved on to the following conceptual topic. The first question was straightforward and straightforward for the pupils. I clarified the question after checking the replies.

   Find an angle θ that makes the trigonometric expression sin (θ) = cos (3θ -10) right.
   A. 25° B. 30° C. 75° D. 60°

2. The right answers were recorded by 40% of the students in the second question. Before I could clarify, we went on to the next round of peer debate. The right answers rose to 95% after the peer discussion.

   Find an angle θ that makes the trigonometric expression tan θ = cot (θ/2 + π/12) true.
   A. 5π / 8 B. 5π / 6 C. 6π / 5 D. 5π / 12

3. The right answers were reported by 55% of the students in the third question. We next progressed to the next level of peer discussion and recorded the responses once more. As with the preceding question, after peer debate, the right answers increased to 90% of the class. After that, I explained the lesson to the students.

   Evaluate the cosecant function cosecant (5π / 6).
   A. π/8 B. π/6 C. π/12 D. π/2

4. Only 45% of the students got the correct answer to the last concept question. Then it was time for peer discussion. The number of right responses increased to 85 percent of the class after peer discussion. After that, I conveyed the matter to the pupils.

   Evaluate the cosecant function cosecant (5π / 6).
   A. 4 B. 1 C. 2 D. 6

Sample Questions

1. Find an angle θ that makes the trigonometric expression sin (θ) = cos (3θ -10) right.

   Solution
   Since we want cofunction values to be equal, the two angles must be complementary.

   Therefore, θ + (3θ - 10°) = 90°

   4θ - 10° = 90°

   Hence, θ = 25°
   The angles θ that makes the expression true is θ = 25°.

2. Find an angle θ that makes the trigonometric expression tan θ = cot (θ/2 + π/12) true.

   Solution

   Again, the two angles must be complementary.

   Hence, θ + (θ/2 + π/12) = π/2

   Therefore, 3θ/2 = π/2 – π/12 = 5π/12 3θ/2 = 5π/12

   Finally, θ = 10π/36 = 5π/18

   The final value of θ = 5π/18.

3. If cos (π/2 – u) = sin (π/8), find the value of variable u given that it lies between 0 and π/2.
   Solution
   Recall the cofunction identity for cosine and use it to assess the given trigonometric expressions.

   cos (π/2 – u) = sin (u)

   Therefore, cos (π/2 – u) = sin (π/8)

   Hence, u = π/8

   Therefore, the value of the variable u is π/8.

4. Evaluate the cosecant function cosecant (5π / 6).

   Solution
   Simplify the given cosecant function by transforming it to an equation with its basic equivalent which is sine.

   Therefore, csc (5π / 6) = 1 / sin (5π / 6)

   Apply the cofunction identity for sine.
   csc (5π / 6) = 1 / sin (π / 2 + π / 3)
   Further simplify the expression and solve for the function.

   csc (5π / 6) = 1 / sin (π / 2 – (-π / 3))

csc (5π / 6) = 1 / cos (-π / 3)

Hence, csc (5π / 6) = 1 / cos (π / 3)

csc (5π / 6) = 2
The value of csc (5π / 6) is 2.

Sample Lesson 3: Verifying Trigonometric Identities

Proofs

1. Prove the identity cot(x) / csc(x) = cos(x)

   cot(x) / csc(x) = [cos(x) / sin(x)] / [1 / sin(x)]
   [cos(x) / sin(x)] / [1 / sin(x)] = [cos(x) / sin(x)] * [sin(x) / 1]
   [cos(x) / sin(x)] * [sin(x) / 1] = cos(x) / 1 = cos(x)

   Then my proof of the identity is all of these steps, put together:

   cot(x) / csc(x) = [cos(x) / sin(x)] / [1 / sin(x)] = [cos(x) / sin(x)] * [sin(x) / 1] = cos(x)

2. Prove the identity cot(x) + tan(x) = sec(x)csc(x)

   cot(x) + tan(x) = cos(x) / sin(x) + sin(x) / cos(x)
   cos(x)/sin(x) + sin(x)/cos(x) = cos^2(x)/sin(x)cos(x) + sin^2(x)/sin(x)cos(x)
   cos^2(x)/sin(x)cos(x) + sin^2(x)/sin(x)cos(x) = [cos^2(x) + sin^2(x)]/sin(x)cos(x)

   Looking back at the RHS of the original identity, I notice that this denominator could be helpful. I'll split the product into two fractions:
   [cos^2(x) + sin^2(x)] / sin(x)cos(x) = 1 / sin(x)cos(x)

   And finally,
   1 / sin(x)cos(x) = [1 / sin(x)] * [1 / cos(x)]

   [1 / sin(x)] * [1 / cos(x)] = csc(x) * sec(x)

   Method

1. After the students' correct answers exceeded 75% on the first question, we moved on to the following conceptual topic. The first question was straightforward and straightforward for the pupils. I clarified the question after checking the replies.

   (1 - sin A)/ (1 + sin A)

2. The right answers were recorded by 50% of the students in the second question. Before I could clarify, we went on to the next round of peer debate. The right answers rose to 95% after the peer discussion.

Prove that, √ {(sec θ – 1)/ (sec θ + 1)}

3. The right answers were reported by 55% of the students in the third question. We next progressed to the next level of peer discussion and recorded the responses once more. As with the preceding question, after peer debate, the right answers increased to 90% of the class. After that, I explained the lesson to the students.

   $\tan^4 \theta + \tan^2 \theta$

4. Only 10% of the students got the correct answer to the last concept question. Then it was time for the topic explanation.

   $\cos \theta / (1 - \tan \theta) + \sin \theta / (1 - \cot \theta)$

Questions

1. $(1 - \sin A)/ (1 + \sin A)$
   A. $(\sec A + \tan A)^2$ B. $(\tan A - \sec A)^2$ C. $(\sec A - \tan A)^2$ D. $(\csc A - \tan A)^2$

Solution

L.H.S = $(1 - \sin A)/ (1 + \sin A)$

= $(1 - \sin A)^2/ (1 - \sin A) (1 + \sin A)$, Multiply both numerator and denominator by $(1 - \sin A)$

= $(1 - \sin A)^2/ (1 - \sin^2 A)$

= $(1 - \sin A)^2/ (\cos^2 A)$, [Since $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$]

= $\{(1 - \sin A)/\cos A\}^2$

= $(1/\cos A - \sin A/\cos A)^2$

= $(\sec A – \tan A)^2$

2. Prove that, $\sqrt{\{(\sec \theta – 1)/(\sec \theta + 1)\}}$
   A. $\operatorname{cosec} \theta - \cot \theta$ B. $\sec \theta - \cot \theta$ C. $\cot \theta - \operatorname{cosec} \theta$ D. $\cos \theta - \cot \theta$

Solution

L.H.S.= $\sqrt{\{(\sec\theta - 1)/(\sec\theta + 1)\}}$

= $\sqrt{[\{(\sec\theta - 1)(\sec\theta - 1)\}/\{(\sec\theta + 1)(\sec\theta - 1)\}]}$; [multiplying numerator and denominator by (sec θ - l) under radical sign]

= $\sqrt{\{(\sec\theta - 1)^2/(\sec^2\theta - 1)\}}$

= $\sqrt{\{(\sec\theta - 1)^2/\tan^2\theta\}}$; [since, $\sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$]

= $(\sec\theta - 1)/\tan\theta$

= $(\sec\theta/\tan\theta) - (1/\tan\theta)$

= $\{(1/\cos\theta)/(\sin\theta/\cos\theta)\} - \cot\theta$

= $\{(1/\cos\theta) \times (\cos\theta/\sin\theta)\} - \cot\theta$

= $(1/\sin\theta) - \cot\theta$

= $\operatorname{cosec}\theta - \cot\theta$

3. $\tan^4\theta + \tan^2\theta$
   A. $\sec^6\theta - \sec^2\theta$ B. $\sec^4\theta - \sec^2\theta$ C. $\sec^4\theta + \sec^2\theta$ D. $\sec^4/\sec^2\theta$

Solution

L.H.S = $\tan^4\theta + \tan^2\theta$

= $\tan^2\theta(\tan^2\theta + 1)$

= $(\sec^2\theta - 1)(\tan^2\theta + 1)$ [since, $\tan^2\theta = \sec^2\theta - 1$]

= $(\sec^2\theta - 1)\sec^2\theta$ [since, $\tan^2\theta + 1 = \sec^2\theta$]

= $\sec^4\theta - \sec^2\theta$

4. $\cos\theta/(1 - \tan\theta) + \sin\theta/(1 - \cot\theta)$
   A. $\sin\theta + \cos\theta$ B. $\sin\theta - \cos\theta$ C. $\sin\theta/\cos\theta$ D. $\cos\theta + \cos\theta$

Solution

L.H.S = $\cos\theta/(1 - \tan\theta) + \sin\theta/(1 - \cot\theta)$

$= \cos\theta/\{1 - (\sin\theta/\cos\theta)\} + \sin\theta/\{1 - (\cos\theta/\sin\theta)\}$

$= \cos\theta/\{(\cos\theta - \sin\theta)/\cos\theta\} + \sin\theta/\{(\sin\theta - \cos\theta/\sin\theta)\}$

$= \cos^2\theta/(\cos\theta - \sin\theta) + \sin^2\theta/(\cos\theta - \sin\theta)$

$= (\cos^2\theta - \sin^2\theta)/(\cos\theta - \sin\theta)$

$= [(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)]/(\cos\theta - \sin\theta)$

$= (\cos\theta + \sin\theta)$

2.3 RESULTS

Data Analysis

In the study, item analysis was performed using SPSS 21.0 program in order to calculate the reliability of the mathematics achievement test prepared by the researcher regarding "statistics (table, graphs, arithmetic mean, openness)".

During the research, after the 10-week implementation phase, the collected data were analyzed. Statistical calculations were made on 171 primary school 9th grade students, 69 of whom were experimental and 102 were control group. In order to examine the effect of Peer Education on academic accomplishment and attitude in mathematics lesson, the achievement test and attitude scale were applied at two different times as pretest and posttest. In the final stage of the application The Peer Instruction Student Evaluation Form was applied.

T-test was used to compare treatment and control groups. The significance of the difference between the mean scores of the groups was interpreted at the 0.05 level.

Normality of Mathematics Achievement Test

Before selecting the statistical test required to examine the pretest scores of the learners in the treatment and control groups, it is necessary to examine whether the pretest scores are normally distributed or not.

Table 6 - Mathematics Achievement Pretest Skewness and Kurtosis Values

	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
Pretest	3.338	0.827	0.186	0.462	0.369
Aksay	3.451	0.477	0.271	-0.668	0.535
Sdk	1.914	-0.307	0.361	-1.150	0.709

Girls	2.207	0.121	0.340	-0.169	0.668

Table 7 - *Mathematics Achievement Posttest Skewness and Kurtosis Values*

	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
Posttest	4.024	0.142	0.186	-0.773	0.369
Aksay	3.804	-0.288	0.271	-0.429	0.535
Sdk	3.767	0.313	0.361	-0.910	0.709
Girls	3.832	0.773	0.340	0.152	0.668

The normality of both scores is supported as seen in the table since the skewness and kurtosis values are between -1 and +1 as reported by L.F. Hair, W. C. Black, B. J. Babin, R. E. Anderson and R. L. Tatham [273].

The distortion coefficient in the normal distribution is another factor. If the coefficient of skewness is "0", the total symmetric distribution according to the average, it is negative (0) to be less than 0, and positive (right) to be greater than 0 indicates the distortion. If the skew coefficient remains within ± 1, it can be interpreted that the scores do not show a significant deviation from the normal distribution. It is also important to calculate the kurtosis measure so that the distribution can be considered normal. If the kurtosis is "3", the normal distribution will be, if it is less than 3, the series will be flat, and if it is larger than 3, the series will be sharp [274]. The kurtosis and skewness values indicate whether the data show a normal distribution. Since the normal distribution is symmetrical, the arithmetic mean, median and modes of the variables with normal distribution are equal. If a distribution is not symmetrical, the peak of the bell curve in the distribution curve will be shifted to the right or to the left, not in the middle. If the subjects are gathered at values greater than the average, the left-skewed distribution, if they are gathered at small values, the right-skewed distribution is mentioned. Average > median> mode order in right-sided distributions; For left-sided distributions, mode> median> average rank can usually be made [275]. In this case, it can be concluded that the success of the group is low since the majority of the positive (positive) points are collected below the mean in the right-skewed distribution and the success of the group is high in the left-skewed (negative) distribution.

In addition, the two histograms shown below support pretest and posttest average scores normality of the mathematics achievement test.

Figure 3 - Mathematics achievement test pretest scores' histogram

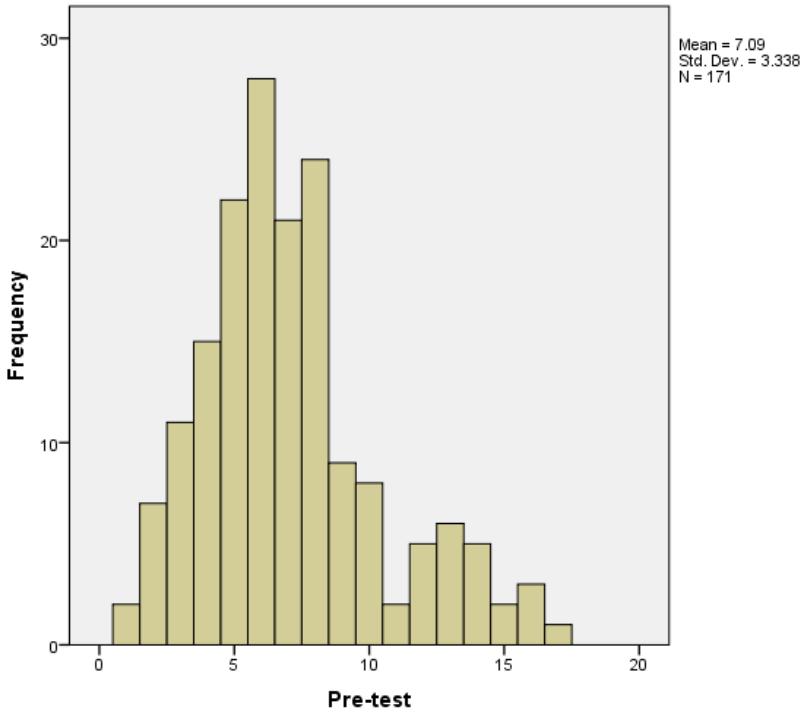
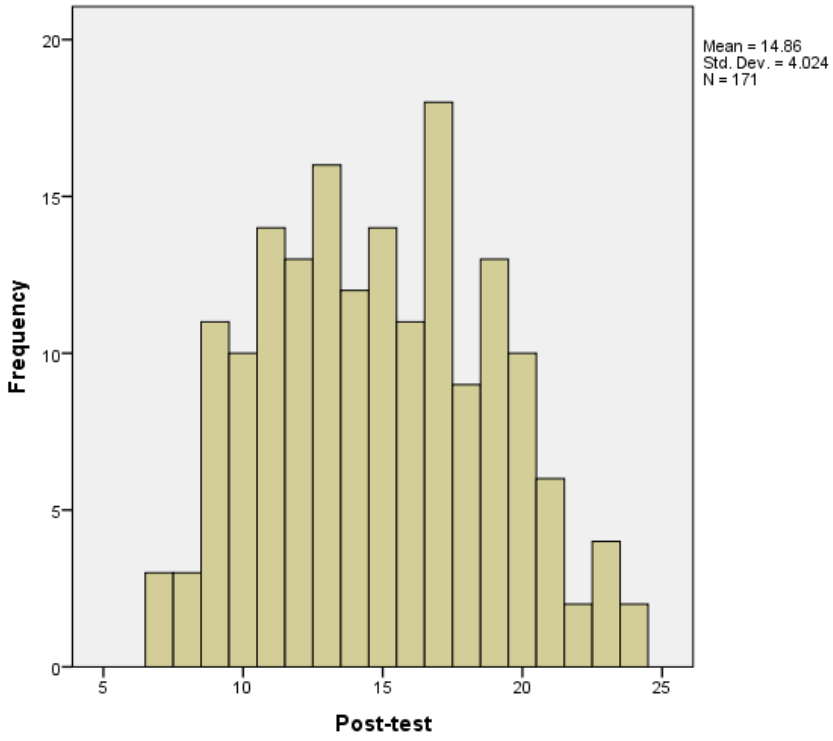


Figure 4 - Mathematics achievement test posttest scores' histogram



Normality of the Attitude towards Mathematics Scale (ATMS)

In this research, initially, the normality of the scores was examined after the total attitude scores were measured. In the present research, as illustrated above, the scores were normally distributed and the number of the participants in both groups

were bigger than 30. The skewness and kurtosis values of each school and the distribution of the scores are as shown in the table below.

Table 8 - Pretest Skewness and Kurtosis Values of The Attitude towards Mathematics Scale

	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
Pretest	2.778	0.354	0.186	0.028	0.369
Aksay	2.029	0.194	0.271	0.025	0.535
Sdk	2.781	-0.152	0.361	-0.469	0.709
Girls	3.500	0.187	0.340	-0.480	0.668

Table 9 - Posttest Skewness and Kurtosis Values of The Attitude towards Mathematics Scale

	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Std. Error
Posttest	3.046	0.543	0.186	0.616	0.369
Aksay	2.979	0.082	0.271	-0.670	0.535
Sdk	2.756	0.587	0.361	0.310	0.709
Girls	3.221	1.209	0.340	1.450	0.668

The normality of both scores is supported as seen in the table since the skewness and kurtosis values are between -1 and +1 as L.F. Hair, W. C. Black, B. J. Babin, R. E. Anderson, and R. L. Tatham, we can state that the pre and post attitude scores were normally distributed. Moreover, the given figures below support the total attitude scores' normality [273, p. 46].

Figure 5 - Histogram of pretest scores of The Attitude towards Mathematics Scale

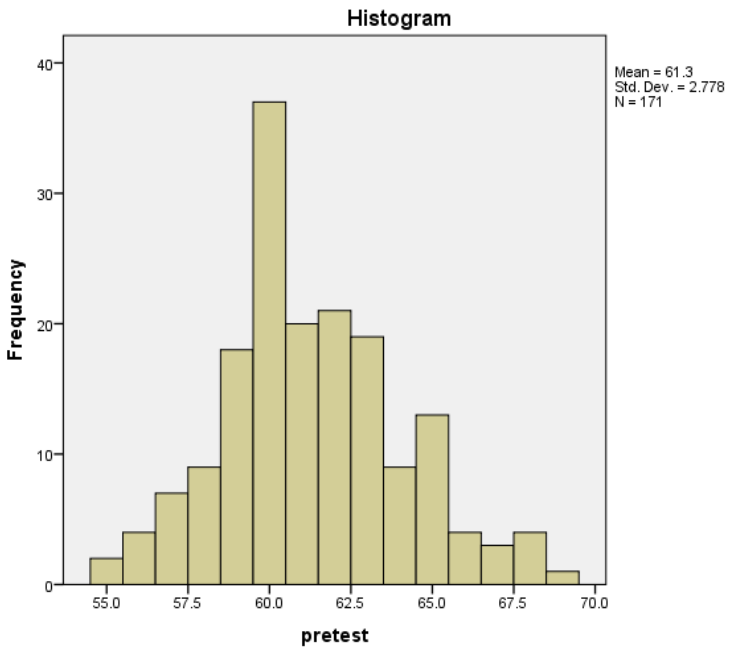
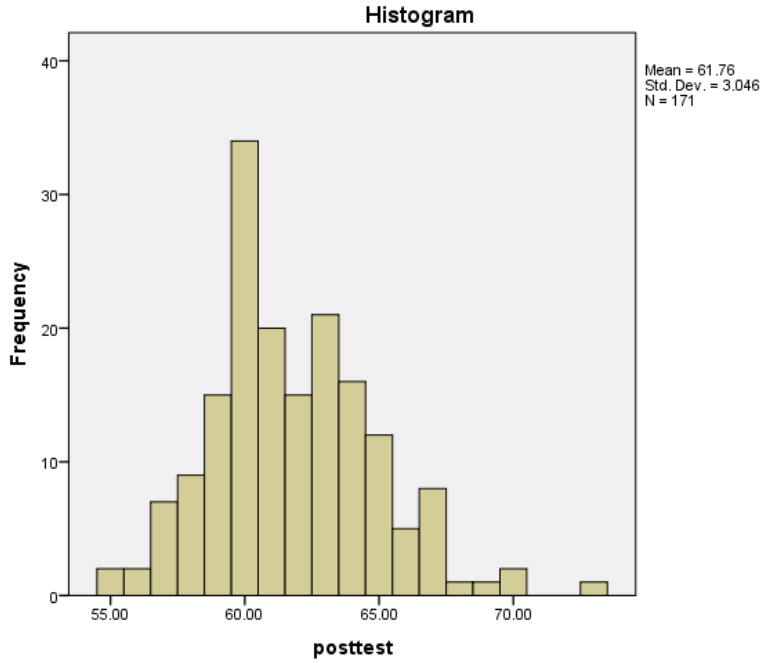


Figure 6 - Histogram of posttest scores of The Attitude towards Mathematics Scale



The Finding of Mathematics achievement test (MAT)

1. Sub-question: Is there a significant difference in the pretest achievement scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied?

To find out whether there is an important difference between the treatment group in which peer instruction is applied and the pretest average of scores of academic an accomplishment of the control group learners who use the traditional teaching method, the Independent Sample T-test was used. The analysis of the group statistics of the treatment and control group students from the pretest is given in Table 10.

Table 10 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Pretest	Experimental	69	6.55	2.893	.348
	Control	102	7.45	3.576	.354

Table 10 is a total analysis of the group statistical data acquired from the pretest scores of mathematics achievement tests in treatment and control groups. As indicated from the table, the treatment group's pretest average score in the

mathematics achievement test is 6.55 (SD = 2.893) while from the same test, the control group's pretest average score is 7.45 (SD = 3.576). Equivalence of pretest scores to each other is an important factor in determining and interpreting independent variables clearly. With a similar result in this study, the assumption is that the effectiveness of the instructional methods used will be determined more accurately.

The pretest mathematics accomplishment scores of the students in the treatment and control groups calculated in Excel are shown in Figure 7.

Figure 7 - Pretest Success Score Average of Experimental and Control Groups

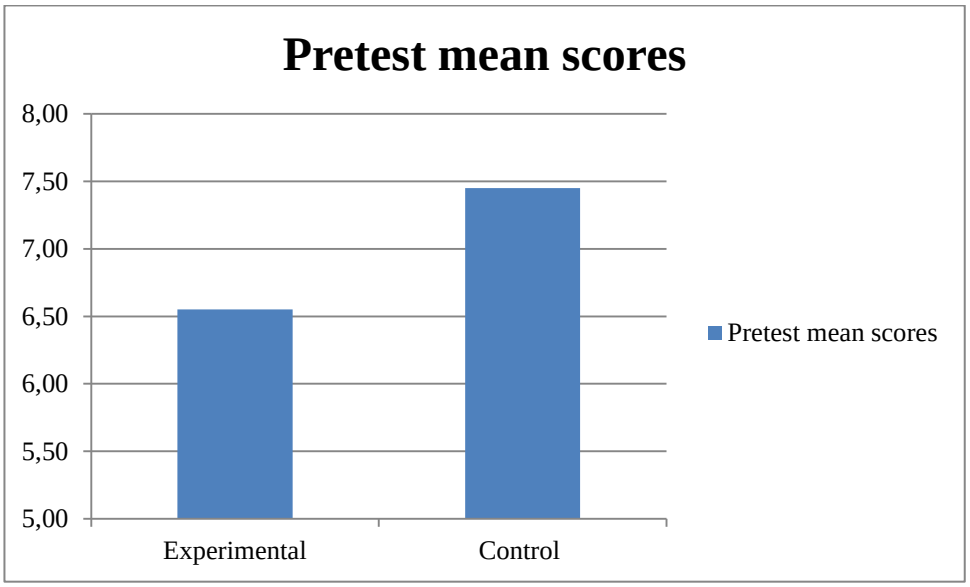


Table 11 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal	2.1	.15	-	169	.0	-.90	.52	-1.92	.12

	varianc es assume d	4		1.74		8				
	Equal varianc es not assume d			- 1.81	163. 56	.0 7	-.90	.50	-1.89	.08 0

When we consider the results of the statistical Independent Sample T-Test prepared to understand if there was a considerable distinction between average scores of the pretest scores of the treatment and the control groups, an important distinction was not found between the average of the scores of the two groups ($p = .084$; $p > 0.05$). Based on these outcomes, it can be assumed that the success scores of the treatment and control group learners before practice are equivalent.

2. Sub-question: Is there a significant difference in students' achievement mean scores between pretest and posttest scores in the experimental group where peer instruction is applied?

To examine if there was a considerable distinction between the mathematics achievement test pretest and posttest average results of the treatment group students in which peer education was applied, the Independent Sample T-test was used. Review of the relationship between the treatment group students' achievement test pretest and posttest average of the scores are given in Table 12.

Table 12 - Group Statistics

	Tests	N	Mean	Std. Deviation	Std. Error Mean
Experiment al	Pretest	69	6.55	2.893	.348
	Posttest	69	17.14	3.465	.417

Table 12 is a total analysis of the descriptive statistics collected from the pretest scores and posttest scores of mathematics achievement tests in the treatment group. As indicated from the table, the treatment group’s pretest average score in the mathematics achievement test is 6.55 (SD = 2.893). On the other hand, the posttest average of the score in the same test is 17.14 (SD = 3.465).

The average mathematics achievement score of the experimental group students after the application was higher than the average mathematics achievement scores before the application.

This situation is shown in Figure 8.

Figure 8 - Experimental Group's Pretest and Posttest Success Score Averages

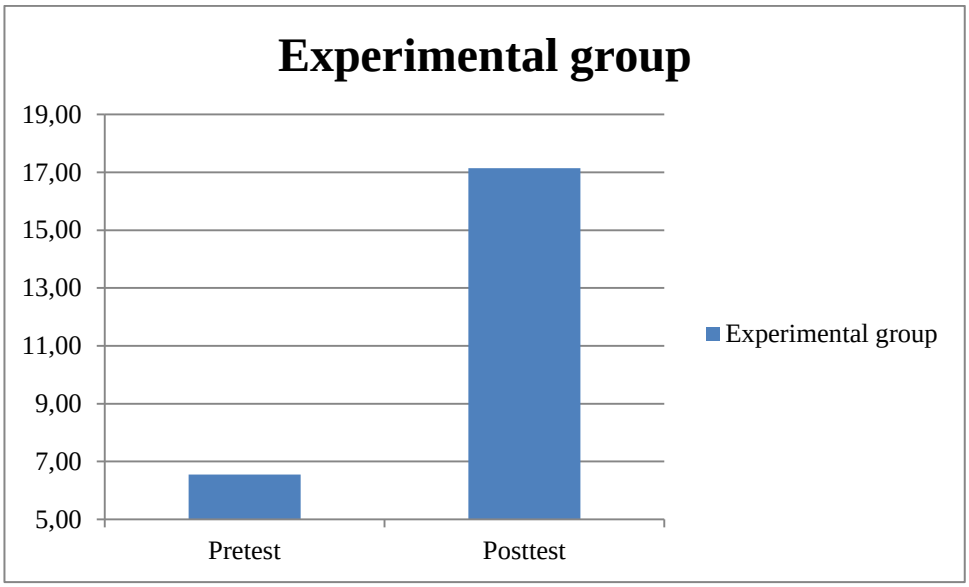


Table 13 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	3.36	.07	-19.50	136	.00	-10.59	.54	-11.67	-9.52
	Equal variances not assumed			-19.50	131.80	.00	-10.59	.54	-11.67	-9.52

According to the analysis results, there is an important difference between the pretest and posttest average scores of the learners who receive peer instruction ($p = .000$; $p < 0.05$). When we look at the average rank and cumulative total of the difference in the scores, it is seen that this observed difference is in favor of the posttest score. According to these results, it can be said that peer education has an important effect on increasing students' achievement in statistics in mathematics lessons.

3. Sub-question: Is there a significant difference in students' achievement mean scores between pretest and posttest scores in the control group where traditional teaching is applied?

In order to examine whether there was a considerable difference between the mathematics achievement test pretest and posttest average scores of the treatment group students in which traditional teaching is applied, the independent Sample T-test was used. Analyzes of the relationship between the control group students' achievement test pretest and posttest average scores are given in Table 14.

Table 14 - Group Statistics

	Tests	N	Mean	Std. Deviation	Std. Error Mean
Control	Pretest	102	7.45	3.576	.354
	Posttest	102	13.31	3.631	.360

Table 14 is a representation of the general analysis of the data derived from the pretest average scores and posttest average scores of mathematics achievement tests in the control group. The experimental group's pretest average score in the mathematics achievement test score as can be seen from the table is 7.45 (SD = 3.576). On the other hand, the posttest average of the score in the same test is 13.31 (SD = 3.631).

The average scores of mathematics achievement of the control group students after the application were higher than the average mathematics achievement scores before the application (Figure 9).

Figure 9 - Pretest and Posttest Success Score of the Control Group

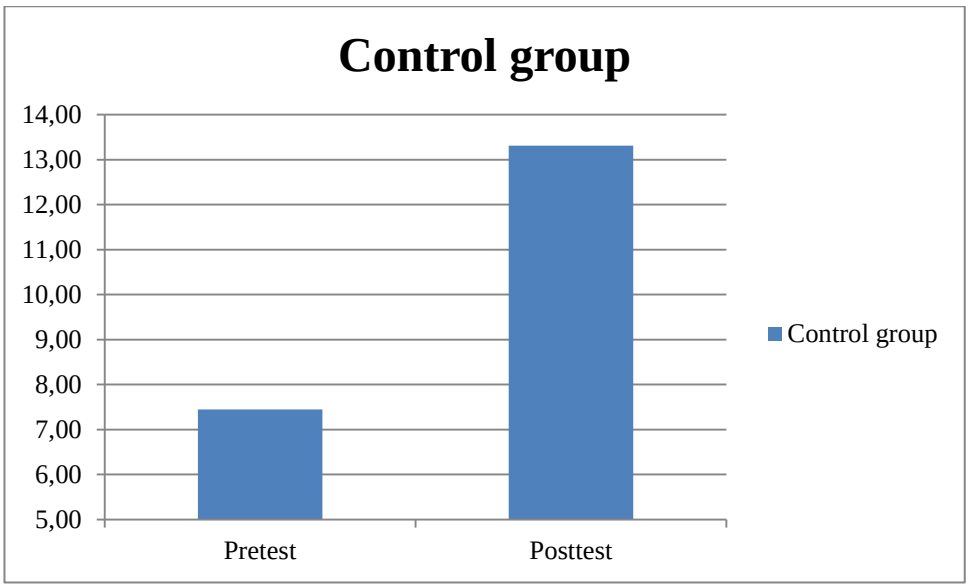


Table 15 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	.91	.341	-11.62	202	.00	-10.59	5.86	-6.86	-4.87
	Equal variances not assumed			-11.62	201.95	.00	-10.59	5.86	-6.86	-4.87

According to the analysis results, we noted an important difference in the pretest and posttest average scores of the students who receive peer instruction (p = .000; p

<0.05). Considering the average rank and total of the difference scores, it is seen that this observed distinction is in favor of the posttest score. According to these results, it can be said that traditional teaching has a meaningful effect on increasing students' achievement in statistics in mathematics lessons.

4. Sub-question: Is there a significant difference in the posttest achievement scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied?

When examining whether there is an important difference between the treatment group of students in which peer instruction is applied and the posttest mean scores of the control group of students who used the traditional teaching method, the Independent Sample T-test was used. The analysis of the average scores of the treatment and control group students from the pretest is given in Table 16.

Table 16 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Posttest	Experimental	69	17.14	3.465	.417
	Control	102	13.31	3.631	.360

Table 16 is a posttest scores analysis of mathematics achievement retrieved from the study conducted in treatment and control groups. Looking at the table, the treatment group's posttest average of the score in the mathematics achievement test is 17.14 (SD = 3.465). On the other hand, the control group's posttest average of the score in the same test is 13.31 (SD = 3.631).

The posttest mathematics accomplishment scores of the students in the treatment and control groups are shown in Figure 10.

Figure 10 - PostTest Success scores of Experimental and Control Groups

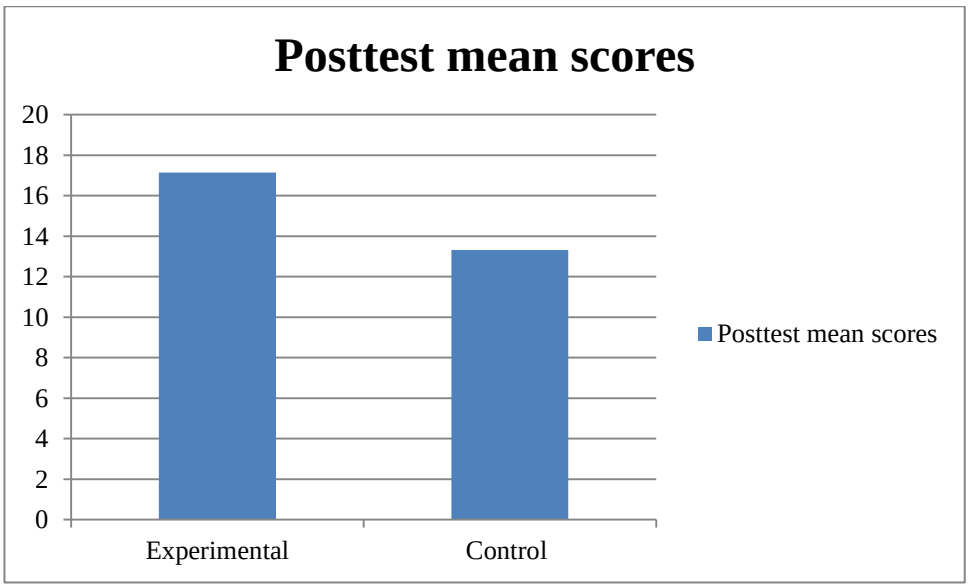


Table 17 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	.345	.558	6.90	169	.00	3.83	.56	2.73	4.93
	Equal variances not assumed			6.90	150.61	.00	3.83	.55	2.73	4.92

After the T-test results obtained for understanding whether there is an important difference between the posttest scores of the treatment and control groups were analyzed, an important disparity was found between the mean scores of the two groups ($p = .000$; $p < 0.05$). Based on this analysis, it cannot be said that the

mathematics accomplishment scores of the treatment and control group learners after practice are equivalent.

The Finding of Attitudes towards Mathematics Scale (ATMS)

5. Sub-question: Is there a significant difference in students' attitudes mean scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied in pretests scores?

The Independent Sample T-test was used to examine whether there is a meaningful distinction between the treatment group in which peer instruction is applied and the pretest mean scores of attitudes towards mathematics of the control group students who use the traditional teaching method. The analysis of the scores of the treatment and control group students from the pretest is given in Table 18.

Table 18 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Pretest	Experimental	69	3.08	.151	.018
	Control	102	3.06	.130	.013

Table 18 is a total analysis of the statistical data collected from the pretest scores of attitudes towards mathematics in treatment and control groups. As given in table 18, the treatment group's pretest average of the score in the attitudes survey is 3.08 (SD = .151). On the other hand, the control group's pretest average score in the same test is 3.06 (SD = .130). Equivalence of pretest scores to each other is an important factor in determining and interpreting independent variables clearly. With a similar result in this study, it's assumed that the effectiveness of the instructional techniques used will be determined more accurately.

In other words, it can be said that the attitudes of the two groups towards the mathematics lesson were equal before the implementation (Figure 11).

Figure 11 - Pre Attitude Score Average of Experimental and Control Groups

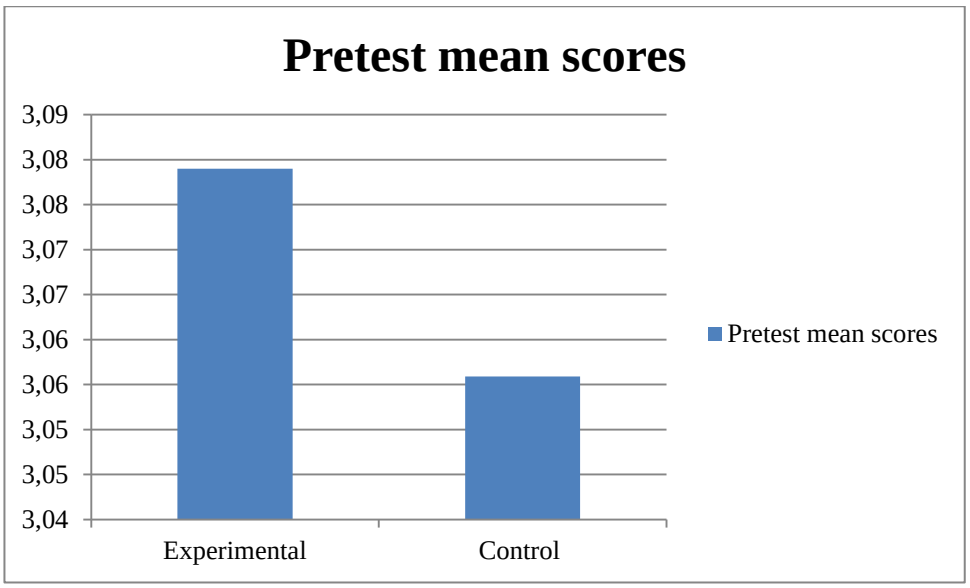


Table 19 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	2.84	.094	1.07	169	.29	.023	.021	-.02	.07
	Equal variances not assumed			1.04	131.34	.30	.023	.022	-.02	.07

When we consider the analysis of the t-test conducted on the if there is no important disparity between the pretest scores of the treatment and control groups, a significant disparity was not found between the average of the scores of the two

groups (p = .287; p > 0.05). Based on this analysis, it can be assumed that the attitudes towards mathematics of the experimental and control group learners before practice are equivalent.

6. Sub-question: Is there a significant difference in students' attitudes mean scores between pretest and posttest scores in the experimental group where peer instruction is applied?

The independent sample T-test was used to examine whether there was an important distinction between the attitudes towards mathematics survey test pretest and posttest mean scores of the treatment group students in which peer education was applied. Analysis of the relationship between the treatment group students' attitudes tests pretest and posttest mean scores are given in Table 20.

Table 20 - Group Statistics

	Tests	N	Mean	Std. Deviation	Std. Error Mean
Experimental	Pretest	69	3.08	.151	.018
	Posttest	69	3.65	.243	.029

Table 20 is a group statistical analysis made from the total pretest scores and posttest scores of attitudes towards survey tests in the treatment group. As indicated by the table, the treatment group's pretest average score in the attitude towards mathematics test is 3.08 (SD = .151). On the other hand, the posttest average score in the same test is 3.65 (SD = .243).

The average scores of the mathematics attitude test of the treatment group students after the application were higher than the average of the attitude test before the application (Figure 12).

Figure 12 - The Experimental Group's Pre-Attitude and Post-Attitude Score Averages

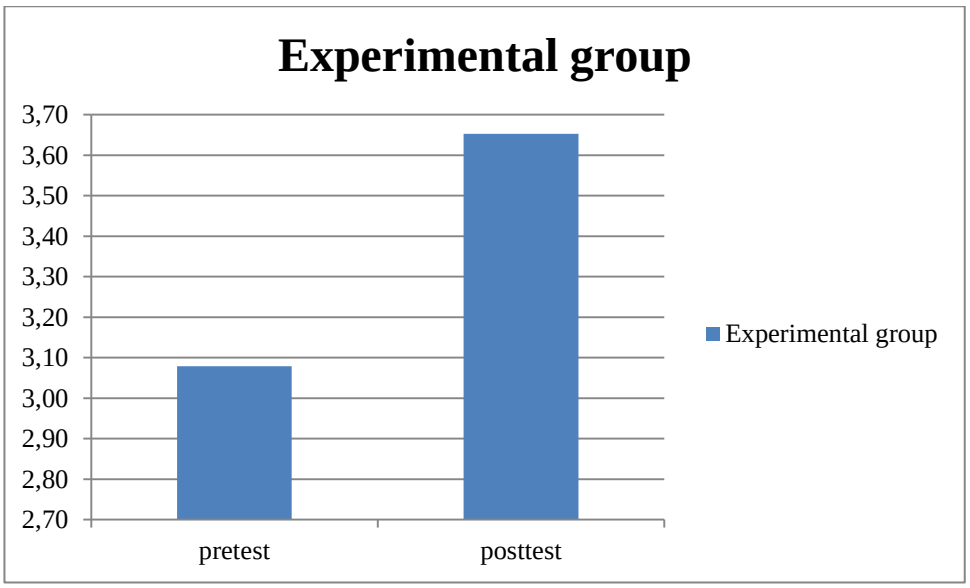


Table 21 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	24.44	.000	-16.69	136	.00	-.57	.034	-.64	-.51
	Equal variances not assumed			-16.69	113.72	.00	-.57	.034	-.64	-.51

According to the independent samples T-test analysis results, there is an important distinction between the pretest and posttest average scores of the students who receive peer instruction ($p = .000$; $p < 0.05$). Taking into consideration the mean rank

and total of the disparity scores, it was determined that this disparity is in favor of the posttest score. According to these results, it can be said that peer education has an important effect on increasing students' attitudes in statistics in mathematics lessons.

7. Sub-question: Is there a significant difference in students' attitudes mean scores between pretest and posttest scores in the control group where traditional teaching is applied?

With the aim of determining a meaningful distinction in the attitudes towards mathematics, the average of the pretest and posttest scores of the control group in which peer education was applied was taken and the Independent Sample T-test was applied. Analysis of the relationship between the control group learners' attitudes survey test pretest and posttest average scores are given in Table 22.

Table 22 - Group Statistics

	Tests	N	Mean	Std. Deviation	Std. Error Mean
Control	Pretest	102	3.06	.130	.013
	Posttest	102	3.13	.188	.019

Table 22 is a group statistical analysis made from the total pretest average scores and posttest average scores of attitudes towards mathematics in the control group. As indicated in the table, the control group's pretest average of the score in the attitudes survey test is 3.06 (SD = .130). On the other hand, the posttest average score in the same test is 3.13 (SD = .189).

The average scores of the mathematics attitude test of the control group students after the application were higher than the pre-application attitude test scores (Figure 13).

Figure 13 - The Pre-Attitude and Post Attitude Score Averages of the Control Group

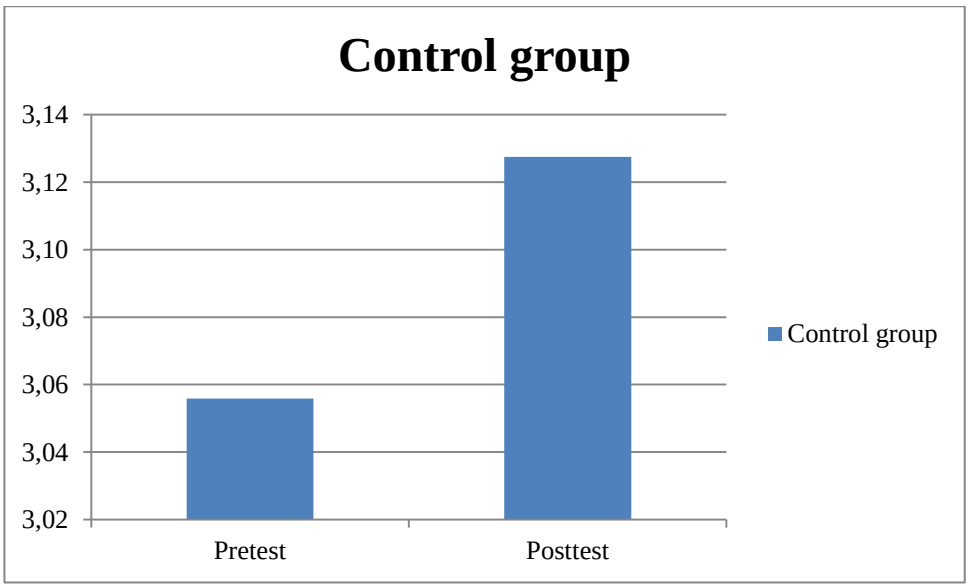


Table 23 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	8.33	.004	-3.16	202	.00	-.07	.023	-.12	-.03
	Equal variances not assumed	..		-3.16	179.50	.00	-.07	.023	-.12	-.03

According to the analysis results, there is an important disparity between the pretest and posttest average scores of the learners who receive peer instruction ($p = .002$; $p < 0.05$). According to these results, it can be said that traditional education has

an important effect on increasing students' attitudes towards mathematics in statistics in mathematics lessons.

8. Sub-question: Is there a significant difference in students' attitudes mean scores between the experimental group where peer instruction is applied and the control group where traditional teaching is applied in posttests scores?

With the aim of determining an important difference in the attitudes towards mathematics, the average of the pretest and posttest scores of the treatment group in which peer education was applied was taken and the Independent Sample T-test was applied. Analysis of the relationship between the control group students' attitudes survey test pretest and posttest average scores are given in Table 24.

Table 24 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Posttest	Experimental	69	3.65	.243	.029
	Control	102	3.13	.188	.019

Table 24 is a cumulative analysis of the statistical data obtained from the posttest scores of attitudes towards mathematics in treatment and control groups. As seen from this table, the treatment group's posttest average of the score in the attitudes survey test is 3.65 (SD = .243). On the other hand, in the exact same test, the control group's posttest average score is 3.13 (SD = .189).

That is, the final attitude scores of the experimental group were higher after the application (Figure 14).

Figure 14 - Final Attitude Score Averages of Experimental and Control Groups

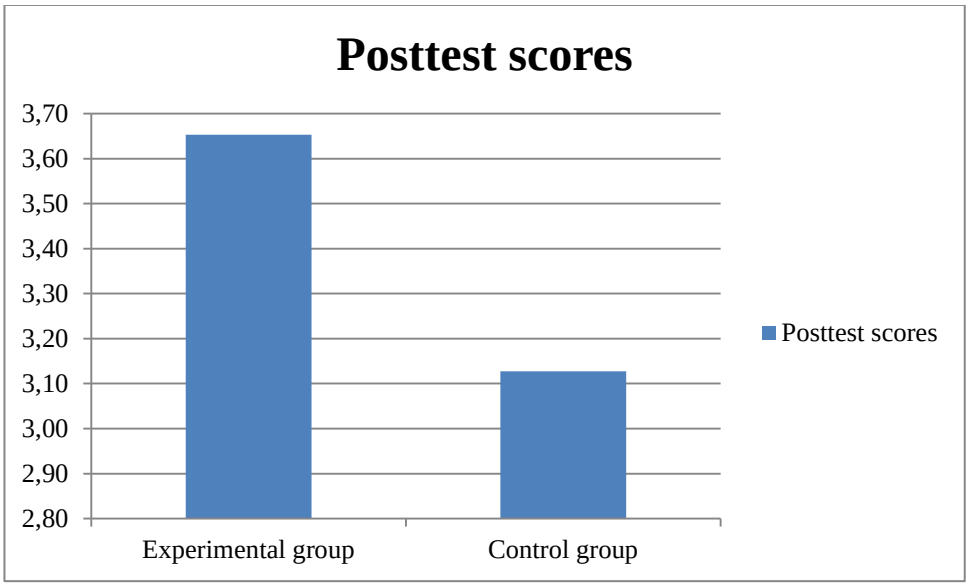


Table 25 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	12.76	.000	15.90	169	.00	.53	.033	.46	.59
	Equal variances not assumed			15.16	121.23	.00	.53	.035	.46	.59

An important distinction was found between the mean scores of the two groups ($p = .000$; $p < 0.05$) as stated by the analysis of the T-Test conducted to understand whether an important disparity was found between the posttest scores of the treatment and control groups. Based on these results, it cannot be said that the attitudes towards mathematics survey scores of the treatment and control group students after practice are equivalent.

The Findings of Gender Differences on Students' Mathematics Achievement

9. Sub-question: Is there a significant difference between the mathematics pretest achievement scores of female and male students in the experimental group where peer instruction is applied?

The independent Sample T-test was used to examine if there is an important difference between the male and female gender groups in which peer instruction is applied to the pretest average scores of academic achievement. The analysis of the scores of the male and female students from the pretest is given in Table 26.

Table 26 - Group Statistics

	Genders	N	Mean	Std. Deviation	Std. Error Mean
Pretest	Male	122	7.69	3.528	.319
	Female	49	5.59	2.207	.315

Table 26 is a cumulative analysis of the statistical data obtained from the pretest scores of mathematics achievement tests in male and female gender groups. As indicated from the table, the male group's pretest average of the score in the mathematics achievement test is 7.69 (SD = 3.528). Conversely, the female group's pretest average score in the same test is 5.59 (SD = 2.207).

The pretest math achievement scores of the students in the male and female groups calculated in Excel are shown in Figure 15.

Figure 15 - Pretest Success Score Average of male and Female Groups

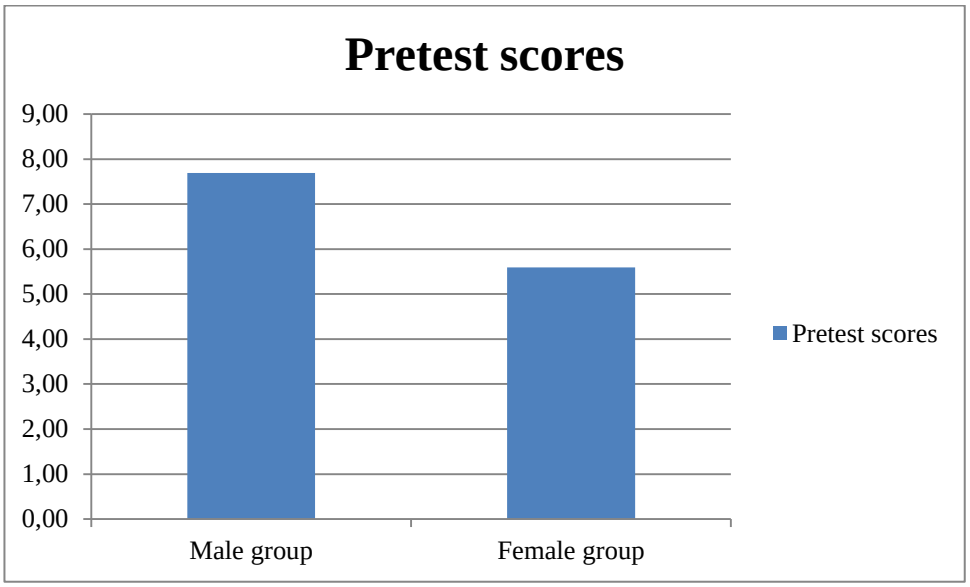


Table 27 - Independent Samples Test

	Levene's Test for Equality of Variances		t-test for Equality of Means					
	F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the

									Difference	
									Lower	Upper
	Equal variances assumed	8.63	.004	3.86	169	.00	2.10	.54	1.03	3.17
	Equal variances not assumed			4.67	139.00	.00	2.10	.45	1.03	2.98

As stated by the analysis of the T-Test conducted to understand whether an important disparity was found between the pretest scores of the male and female gender groups, an important difference between the average scores of the two groups was found ($p = .000$; $p < 0.05$).

10. Sub-question: Is there a significant difference between the mathematics posttest achievement scores of female and male students in the experimental group where peer instruction is applied?

Aiming to determine if there is an inportant difference between the male and female gender groups in which peer instruction is applied to the posttest mean scores of academic accomplishment, an Independent Sample T-test was used to examine. The analysis of the scores of the male and female students from the pretest is given in Table 28.

Table 28 - Group Statistics

	Genders	N	Mean	Std. Deviation	Std. Error Mean
Posttest	Male	122	15.20	4.065	.368
	Female	49	14.02	3.832	.547

Table 28 is a cumulative analysis of the statistical data obtained from the posttest results of mathematics achievement tests in male and female gender groups. As seen from the table, the male group's posttest average score in the mathematics achievement test is 15.20 (SD = 4.065). On the contrary, in the same test, the female group's posttest mean score is 14.02 (SD = 3.832).

The posttest mathematics accomplishment scores of the students in the male and female groups are shown in Figure 16.

Figure16 - Postest Success Score Average of male and Female Groups

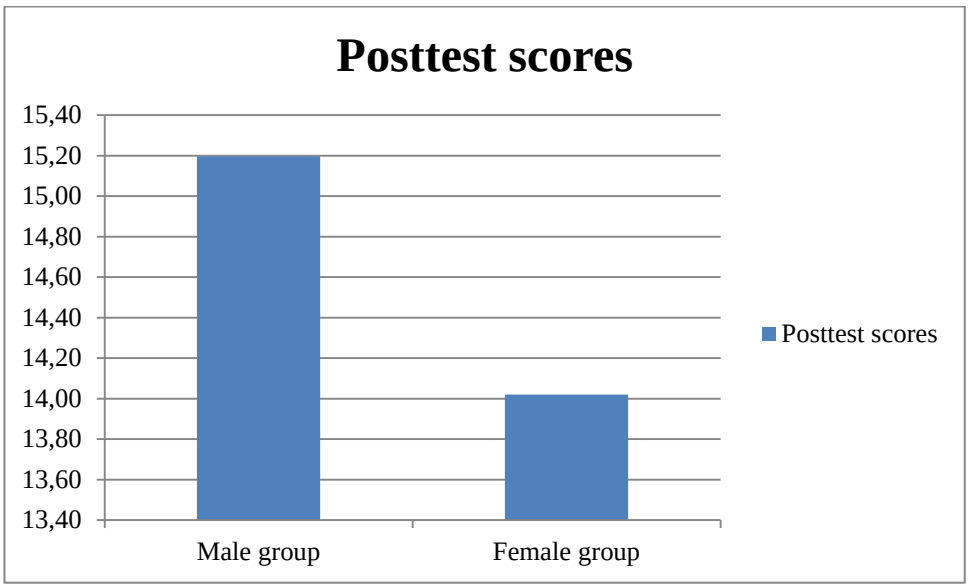


Table 29 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	1.55	.215	1.74	169	.08	1.18	.68	-.16	2.51
	Equal variances not assumed			1.7	93.5	.08	1.18	.66	-	2.49

As determined by the analysis of the T-Test conducted to understand if there is no significant disparity between the posttest scores of the male and female gender groups, a significant disparity was not found between the average of the scores of the two groups ($p = .084$; $p > 0.05$).

The Findings of Gender Differences on Students' Attitudes towards Mathematics

11. Sub-question: Is there a significant difference between the pre-attitude scores of female and male students in the experimental group where peer instruction is applied?

An Independent Sample T-test was used to examine if there is a meaningful disparity between the male and female gender groups in which the traditional method is applied to the pretest mean scores of attitude towards mathematics. The analysis of the scores of the male and female learners from the pretest is given in Table 30.

Table 30 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Pretest	Male	122	3.05	.121	.0110
	Female	49	3.15	.	.

Table 30 is a conclusive statistical analysis gathered from the pretest results of mathematics achievement tests in male and female gender groups. As determined from the table, the male group's pretest average score in the attitude towards mathematics survey test is 3.05 (SD = .121). On the other hand, the female group's pretest average score determined using the same test is 3.15 (SD = .000). Equivalence of pretest scores to each other is an important factor in determining and interpreting independent variables clearly. With a similar result in this study, it is assumed that the effectiveness of the teaching modes used will be determined more accurately.

In other words, it can be said that the attitudes of the two groups towards the mathematics lesson were equal before the implementation (Figure 17).

Figure 17 - Pre Attitude Score Average of Male and Female Groups

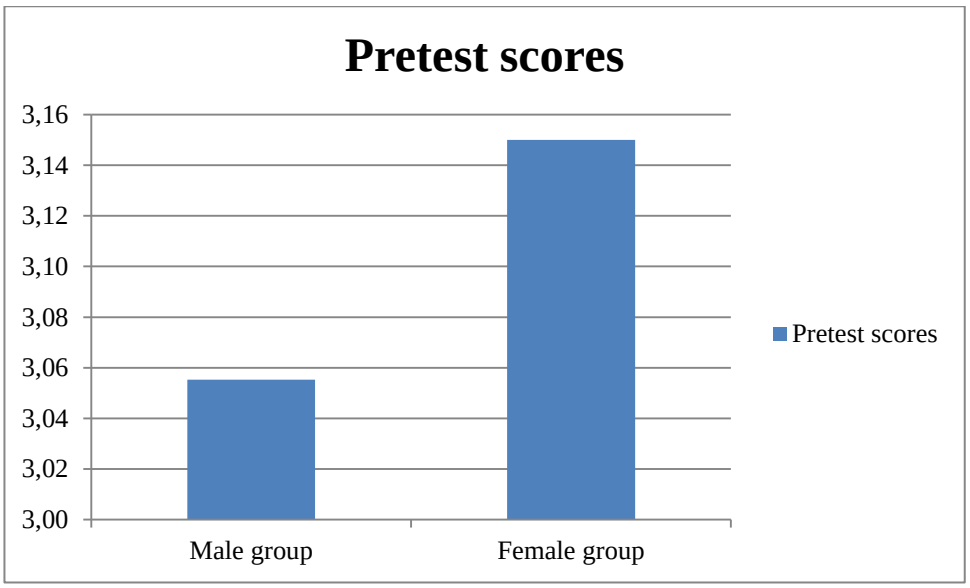


Table 31 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	.	.	-.78	121	.48	-.09	.12	-.33	.15
	Equal variances not assumed			.	.	.	-.09	.	.	.

The analysis of the T-Test conducted to understand if there is no meaningful disparity between the pretest scores of the male and female gender groups indicated

that an important difference was not found between the average of the scores of the two groups (p = .437; p > 0.05).

12. Is there a significant difference between the post-attitude scores of female and male students in the experimental group where peer instruction is applied?

The posttest average scores of attitude towards mathematics of the male and female groups from the traditional lecture method were analyzed using the Independent Sample T-test statistical method with the aim of determining whether there is a difference between the two groups in the traditional method. The analysis of the male and female learners' scores from the pretest is given in Table 32.

Table 32 - Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Posttest	Male	122	3.30	.309	.028
	Female	49	3.44	.352	.050

Table 32 is a statistical analysis collected from the posttest results of mathematics achievement tests in male and female gender groups. As represented in the table, the male group's posttest average score in the attitude towards mathematics survey test is 3.30 (SD = .309). Conversely, obtained from the same test, the female group's posttest mean score is 3.44 (SD = .352).

In other words, the final attitude scores of the male and female groups are close to each other after the application (Figure 18).

Figure 18 - Post Attitude Score Average of Male and Female Groups

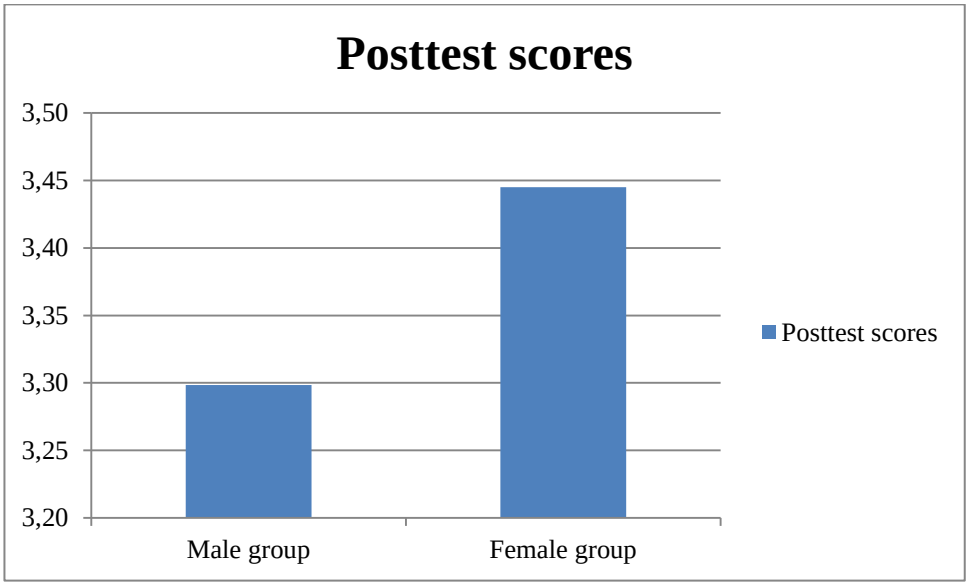


Table 33 - Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	p	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
	Equal variances assumed	2.07	.15	-2.70	169	.00	-.15	.05	-.25	-.04
	Equal variances not assumed			-2.55	79.33	.01	-.15	.06	-.26	-.03

As expressed by the analysis of the T-Test conducted to understand whether there is no meaningful disparity between the posttest scores of the male and female gender groups, an important difference was not found between the average scores of the two groups ($p = .008$; $p > 0.05$).

The Finding of The Peer Instruction student evaluation form

Table 34 - The Peer Instruction student evaluation form

	Strongly Disagree ←---→ Strongly Agree (f)								
Student Evaluations Regarding to the Peer Instruction Method	**1**	**2**	**3**	**4**	**5**	**N**	**Mean**	**Std. Deviation**	**Std. Error**
1. Peer instruction method was clear.		3	6	21	39	69	4.391	0.802	0.160
2. Peer instruction method	1	2	2	27	37	69	4.40	0.890	0.178

was easy to follow.							6		
3. Peer instruction method was interesting.			3	13	53	69	4.725	0.562	0.112
4. Peer instruction method was enjoyable.			7	12	50	69	4.623	0.662	0.132
5. Peer instruction method helped me better understand the course topics.	3	2	1	20	43	69	4.420	0.750	0.150
6. Peer instruction method helped me move beyond my previous level of knowledge	7	10	9	22	21	69	3.580	1.323	0.265
7. Peer instruction method helped me assess my level of knowledge regarding to the course topics.	4	6	11	13	35	69	4.000	1.063	0.213
8. Immediate feedback with the peer instruction method helped me complete my deficiencies.	3	7	15	11	33	69	3.928	1.133	0.227
9. Peer instruction method has increased my confidence in doing course topics.	1	2	12	14	40	69	4.304	0.873	0.175
10. Peer instruction method increased my participation in class.		7	3	11	48	69	4.449	0.753	0.151
11. Peer instruction method increased my motivation towards the course.	12	2	11	24	20	69	3.551	1.161	0.232
12. When I consider all the activities in the course, I think the allocated time for the peer instruction method was sufficient.	1	4	12	22	30	69	4.101	1.194	0.239
13. I think it was difficult to apply the peer instruction method.	32	23		4	10	69	2.087	1.260	0.252
14. I think peer instruction method was useful.	7	4	12	30	16	69	3.638	1.393	0.279
15. I think peer instruction method should be used in other courses as well.	4	17	22	13	13	69	3.203	1.389	0.278

16. I think peer instruction method was educationally attractive.	1	12	3	24	29	69	3.986	0.860	0.172
Student Evaluations Regarding to the Conceptual Questions									
17. The questions posed in the question-answer process of the peer instruction method increased my interest.	9	3	17	20	20	69	3.565	1.198	0.240
18. The questions posed in the question- answer process of the peer instruction method made it easier to understand the important points about the topics.	3	13	2	17	34	69	3.957	1.135	0.227
19. The time allocated for the questions posed in the question-answer process of the peer instruction method was sufficient.	15	13	2	17	22	69	3.261	1.400	0.280
20. The level of difficulty of the questions posed in the question-answer process of the peer instruction method was appropriate for my level.		12	7	31	19	69	3.826	1.167	0.233
Student Evaluations Regarding to the Peer Discussions									
21. The discussion level of the peer instruction was high.	1	4	7	34	23	69	4.072	1.012	0.202
22. I actively participated in discussions during the peer instruction.	3	1	8	24	33	69	4.203	1.071	0.214
23. I liked expressing my ideas during discussions in the peer instruction process.	13	11	7	12	26	69	3.391	1.170	0.234
24. The peer instruction method enabled me being aware of the ideas of my group-mates.	9	11	23	21	5	69	3.029	1.129	0.226

25. I liked to see different perspectives during the peer instruction process.	1	3	7	29	29	69	4.188	1.080	0.216

The outcomes of the statistical analysis of the student evaluation form showed mean scores between 4.725 and 2.145. In the part of Student Evaluations Regarding the peer instruction method, the top mean score was reached from Item 3 (M = 4.725), indicated that 94.5% of the learners trust that the peer instruction approach was interesting. The second maximum average of the score was achieved from Item 4 (M = 4.623), which shows that approximately %92.46 of the students think that the peer instruction technique was enjoyable. Item 10 obtained the third-highest average of the score (M = 4.449), signifying that %88.98 of the participants trusted that peer instruction method increased their participation in class. Items 5 achieved an average score (M = 4.420), and reached a fourth-place ranking, thus revealing that %88.4 of the peer instruction group either agreed or strongly agreed that using the peer instruction method helped them better understand the course topics. Lastly, item 13 got the lowest score mean (M = 2.087), showing that %41.74 of it was thought by participants that applying the peer instruction is hard, which is one of the active learning methods. The second part of the student assessment form Student Assessment Regarding the Conceptual Questions has four items. The scores of items were changed between 3.957 and 3.261. The %79.14-65.22 percentage of peer instruction learners thought that the question-response process of the peer instruction method improved their attention and made it easier to comprehend the significant points about the subjects. Also, they thought that the questions posed in the question-response process of the peer instruction method was sufficient and was appropriate for their level. In the third part "Student Evaluations Regarding the Peer Discussions" there were five items and the items scores were changed between 4.203 and 3.029 with the percentage %84.06-65.8. The views of the students were as follows, they actively attended discussions pending the peer instruction, liked stating their opinion, and liked to see different perspectives. Likewise, they thought that the discussion level of the peer instruction was elevated, the peer instruction technique made it possible for them to be aware of the ideas of my group-mates. In general, the outcomes of the student evaluation form indicate that the peer instruction teaching method is interesting, clear, easy to follow, and enjoyable for the participants. They found that the peer teaching approach increased their motivation and helped to understand subjects. We can see these results in the previous studies (F. Demirel, 2013 [15, p. 70]; T. Gok [17, p. 758]; L. L. Lim [18, p. 39]; L. Porter, C. Bailey-Lee, & B. Simon [177, p. 181]; W. Beekes [178, p. 31]; M. J. Giuliodori, H. L. Lujan and S. E. DiCarlo [202, p.170]; A. Almas, S. Kaymak, O. Nurbavliyev, N. Balta, & K. Kurban [276]). Therefore, the students liked to apply the peer instruction method in mathematics lessons.

2.4 Discussion

In this study, the influence of the Peer Teaching Method on 9th-class students' academic accomplishment on mathematics in the topic trigonometry and their attitudes towards mathematics was investigated. In addition, learners' opinions about the Peer Teaching method were tried to be determined with the Likert type Peer Education Student Assessment form. Furthermore, in the present study, it is proposed to examine the gender gaps regarding mathematics academic accomplishment and attitudes towards mathematics.

Discussion of the Results on Students' Mathematics Accomplishment

First of all, the outcomes of the independent t-test have shown and helped us determine a statistically meaningful impact of peer teaching method on learners' mathematics accomplishment test scores. Accordingly, the Peer Instruction method student groups got importantly higher average scores on the mathematics achievement tests than the learners who were implemented the traditional method. The pretest scores were found in the treatment group ($X = 6.55$) and in the control group ($X = 7.45$), and no statistically important difference was found with the analysis performed by the t-test ($p = .084$, $p > .05$). Equivalence of pretest scores to each other is an important factor in determining and interpreting independent variables clearly. With the similar result in this study, it is thought that the effectiveness of the teaching techniques used will be determined more accurately.

After analyzing the mean of the scores acquired from the academic achievement test, the average mathematics achievement score of the learners in the treatment group before the application was $X = 6.55$, while the average mathematics achievement score after the application was $X = 17.14$. The mean scores of the experimental group learners after the application were higher than the average mathematics achievement scores before the application. According to these results, it can be said that peer education has an important effect on increasing students' achievement in trigonometry in mathematics.

When observing the average of the scores from the achievement test, the average mathematics achievement score of the students in the control group before the application was $X = 7.45$, while the average mathematics achievement score after the application was $X = 13.31$. The mean scores of the students in the control group after the application were higher than the average mathematics achievement scores before the application. According to these results, it can be said that traditional education also increases students' mathematics achievement.

After the examination of the treatment and control groups, it is seen that the success score of the treatment group ($X = 17.14$) is importantly higher than the success score of the control group ($X = 13.31$). This result was found to be statistically significant with the results obtained from the independent t-test ($p = .000$, $p < .005$). From the results, we can observe that in the treatment group, the enhancement between the learners' pretest average scores and posttest average scores is 10.59 while in the control group the increase is 5.86. The fact that the experimental

group students' average mathematics achievement scores are higher than the average scores of the students in the control group can be interpreted as peer education increases their mathematics achievement more. Students who teach with peer education repeated the subject twice while learning and explaining to their group mates. The students who learned the topics they did not understand by asking their friends. Therefore, both sides have benefited from group work. This situation is reflected in the final test scores.

When the relevant literature was examined, it was determined that this result was consistent with the results of the previous studies on the Peer Instruction Method. (C. H. Crouch and E. Mazur [7, p. 975]; F. Demirel [15, p. 88]; Eryilmaz [16, p. 59]; T. Gök [17, p. 757]; G. Akay [23, p. 90]; H. N. McKnight [191, p. 110]; T. Yıldırım and N. Canpolat [198, p. 78]; S. P. Rao and S. E. DiCarlo [204, p. 54]; A. P. Fagen, C. H. Crouch, and E. Mazur [207, p. 209]; R. E. Abdelkarim & E. Abuiyada [244, p. 130]; A. B. Lacaba, J. D. Magalona & T. V. G. Lacaba [245, p. 9]; Oloo, S. N. Mutsotso, & E. N. Masibo [246, p. 14]; Y. Z. Olpak, S. Baltaci, & M. Arican [247, p. 2328]; S. Ouko, C. Aurah, & M. Amadalo [248, p. 179]).

F. Demirel researched the effect of the peer teaching methods on the academic achievement of 6th-grade students and found that peer teaching increased their success in the topic of statistics [15, p. 88]. Likewise, Y. Z. Olpak, S. Baltaci, & M. Arican investigated the effect of peer teaching on the success of primary mathematics teacher candidates in statistics and probability in their study and determined the positive effect of peer education on students' achievements [247, p. 2328]. The same results were reached by T. H. Allison in a study where he investigated the effect of peer education on the success of 8th-grade students in equation and inequality systems [13, p. 98]. Crouch and Mazur (2001) obtained similar results from their studies conducted for 10 years at Harvard University for General Physics 2 courses. C. H. Crouch & E. Mazur stated that Peer Teaching Method increased the success of university students in physics lesson compared to conventional teaching, according to the result of their study at Harvard University, where they taught mechanics subjects with Peer Teaching Method [7, p. 975].

For many decades now, the peer instruction method has been used by educators in teaching large groups of learners and many researchers have elaborated on the significance of peer instruction in education [277]. Active student involvement in learning due to this method might be one of the reasons as to why it has produced good results [278]. As explained in the literature review, learners have the chance to evaluate each other’s work and explain the concepts in a better and simpler way which can be understood easily by their peers since they have close or similar ages or levels (F. Demirel [15, p. 89]; A. P. Fagen, C. H. Crouch, and E. Mazur [207, p. 208]; T. Hooker [278, p. 12]).

Finally, students will be able to do peer assessment and acquire feedback that will improve their understanding. In other words, students don’t have to rely on the teacher when it comes to answering questions or getting feedback. In addition, when

students teach their peers, they feel obligated to making preparation on the topic in advance so that they give effective feedback [279]. These could further explain why students involved in peer instruction got higher grades.

Discussion of the Results on Students' Attitudes towards Mathematics

The other object of present study is to research the impact of peer instruction technique on learners' attitudes towards mathematics lesson. When looking at the average of the scores obtained from the attitude test, the average of the score of the mathematics attitude test before the application was X = 3.08, while the attitude test score average of the control group was found to be X = 3.06. According to these findings, it can be said that the treatment and control group students' attitudes towards the mathematics lesson before the application are equivalent to each other.

The treatment group's pretest average score and the posttest mean score are 3.08 (SD = .151) and 3.65 (SD = .243). When looking at the t-test analysis of the relationship between the attitude test pretest and posttest mean scores of the experimental group students, it is seen that there is a meaningful difference between the pretest and posttest scores of the students who receive peer education ($p = .000$; $p < 0.05$). Based on the analysis results, we can say that peer education has an important effect on increasing students' attitudes in statistics in mathematics lessons.

We obtained the control group's pretest and posttest mean scores from the attitude survey test conducted. The pretest and posttest scores were 3.06 (SD = .130) and 3.13 (SD = .189) respectively. According to the t-test analysis results, we found a significant difference between the pretest and posttest scores of the students who receive peer instruction ($p = .002$; $p < 0.05$). According to these results, it can be seen that traditional education has an effect on increasing students' attitudes towards mathematics in statistics in mathematics lessons.

From the mean scores obtained, there was a meaningful difference ($p = .000$; $p < 0.05$) in the achievement posttest scores of the control (X = 13.31) and treatment (X = 17.15) groups. Moreover, the same observation was made in the posttest scores of the attitudes test of the control (3.13 (SD = .189)) and the experimental (3.65 (SD = .243)) groups. It is stated in the literature that students' having positive attitudes towards the course is one of the factors affecting their learning and academic achievement [280], [281]. Since learners who have a positive attitude towards the course will have higher internal motivation, it is essential to develop and apply teaching techniques that will increase students' attitudes towards the course. From the data collected, we can say that the Peer Instruction method has an important higher impact on students' achievement and attitudes. This factor may be attributed to effective communication and feedback among the students in the classroom. Peer assessment when done effectively, promotes active learning in the classroom through discussions, asking for clarifications, feedback, and making corrections where necessary hence increasing student ability to understand, analyze and evaluate questions. This study proves these facts from the data given and has found that even

though the traditional method has an effect on student achievement, the peer instruction method has an even greater effect on increasing student achievement in mathematics.

A. P. Fagen, C. H. Crouch, and E. Mazur, in their research, the outcome of the findings after statistical analysis from the questionnaires applied to teachers using the peer teaching method is that the method positively changed the atmosphere of the classroom, made the lesson enjoyable, and the students' answers came out as a result of joint decisions made with their peers. Also that they are not afraid of being wrong, student satisfaction and participation are at the highest level [278, p. 14].

According to E. Piepmeier, teaching is not just what the teacher told, but the learning experience that learners share with their peers. Immediate feedback is given to the teacher, and it was observed that the students were more confident in their answers after the part they discussed with their peers. Although it is difficult to ensure that all learners participate actively in the lesson in a crowded classroom, each learner is given an active role every 15 minutes with concept tests in the peer teaching method [282]. The previous studies had similar findings with this present study. In the literature, most of the findings indicated that there is an important affirmative influence of peer education method on attitudes towards mathematics (F. Demirel [15, p. 92]; G. Akay [23, p. 47]; R. Abdelkarim, R. Abuiyada, & S. A. Siddiui [258, p. 1516]; J. B. Campit & R. M. Garin's [14, p. 12]; O. C. Yavuz [259, p. 122]).

The general opinion of the students about the mathematics lesson is that it is difficult, so they cannot do mathematical operations. Such thoughts cause students to increase anxiety and develop negative attitudes towards mathematics [283]. The most important tool to eliminate negative attitudes is knowledge and experience. Students' experiences of mathematics can be shown by causing them to develop positive or negative attitudes towards mathematics [283, p. 138], [284]. Considering that attitudes have the power that guides behavior, a relationship can be mentioned between mathematics achievement and attitude towards mathematics (O. Akdemir [285]; S. M. Uyangör & D. K. Ece [286]; Z. Yücel & M. Koç [287]). In the studies conducted, mathematics comes first among the courses that students fail the most, and one of the reasons for this is determined as students' negative attitudes towards mathematics lesson [288], [289]. Studies have linked the increase in students' success in mathematics lessons to their attitudes towards the lesson [290], [291].

G. Akay examined the influence of the peer instruction methods on the attitudes towards mathematics of 8th-class students and found that peer teaching increased their attitudes on the topic of transformation geometry [23, p. 27]. In another study, R. Abdelkarim, R. Abuiyada, & S. A. Siddiui tested bachelor students' attitudes towards mathematics after the peer instruction method. According to the results, the peer teaching method increased students' attitudes positively [258, p. 1511]. J. B. Campit & R. M. Garin's obtained similar results from their studies conducted for the impact of the Peer Instruction method on the attitudes towards the

mathematics of 30 second-grade college students in the second period of the 2013-2014 academic year [14, p. 13].

Furthermore, mathematics attitude corresponds directly with mathematics achievement as seen from the previous studies as well (F. Demirel [15, p. 93]; G. Akay [23, p. 51]; R. Abdelkarim, R. Abuiyada, & S. A. Siddiui [258, p. 1517). For instance, X. Ma and N. Kishor looked into 113 articles on the relationship between mathematics attitude and mathematics achievement. They concluded that a positive attitude towards mathematics directly relates to good results in mathematics. When we consider the current study, the positive attitudes towards mathematics might have caused a higher success rate while when we consider previous studies, a higher achievement rate can also positively affect mathematics attitudes hence gradually leading to better grades [292].

Discussion of Gender Gaps Regarding Mathematics Success

The outcomes of the independent t-test have reported that there is no statistically important influence of peer instruction technique between gender groups on learners' mathematics accomplishment test scores. In other words, learners who were applied the Peer Instruction method and who applied the traditional method got equivalent scores on the mathematics achievement test. The male group's pretest average of the score in the mathematics accomplishment test is 7.69 (SD = 3.528). On the other hand, the female group's pretest average in the same test is 5.59 (SD = 2.207). Equivalence of pretest scores to each other is an important factor in determining and interpreting independent variables clearly. With the similar result in this study, it is thought that the effectiveness of the teaching methods used will be determined more accurately. When the posttest scores of the male and female groups are examined, the male group's posttest average score in the mathematics accomplishment test is 15.20 (SD = 4.065). Otherwise, the female group's posttest average of the score in the same test is 14.02 (SD = 3.832). According to the results of the independent sample T-Test conducted to understand whether there is no important difference between the posttest average scores of the male and female gender groups, an important difference was not found between the average of the scores of the two groups ($p = .084$; $p > 0.05$). After applying the peer instruction, it is observed that the male group students increased their math achievement test average score from 7.69 to 15.20. Conversely, the female group students increased the achievement test score from 5.59 to 14.02. In summary, the enhancement in male learners' average of the scores from pretest to posttest is 7.51, while the enhancement in female learners' average of the scores from pretest to posttest is 8.43. According to the results, peer instruction affected the success rates of the female group students more. From these results, it can be concluded that peer education affects the success of male and female students equally. In other words, gender has no effect on student achievement.

After analyzing the related studies, the conclusion was that this result was consistent with the outcomes of the previous studies on the Peer Instruction Method.

F. Demirel researched the influence of the peer instructional methods on the academic accomplishment of 6th-grade learners between gender groups and found that peer teaching does not have any effect on academic accomplishment of gender groups [15, p. 91]. In another study G. Akay, examined the effect of the peer teaching methods on the academic accomplishment of 6th-grade learners between gender groups and found that peer teaching does not have any effect on academic achievement of gender groups on the topic of transformation geometry [23, p. 50].

Discussion of Gender Differences Regarding Mathematics Attitude

On top of that, the outcome of the independent t-test has informed that there is no statistically important impact of peer instruction technique between gender groups on students' attitudes towards mathematics test scores. The male group's pretest mean score in the mathematics attitudes test is 3.06 (SD = .121) and the female group's pretest average of the score in the same test is 3.20 (SD = .000). The male group's posttest mean score in the attitude towards mathematics survey test is 3.30 (SD = .309). On the contrary, in the same test, the female group's posttest mean score is 3.45 (SD = .352). As shown by the outcome of the analysis of the results done to understand whether there is no important difference between the posttest scores of the male and female gender groups, an important difference was not found between the average of the scores of the two groups ($p = .008$; $p > 0.05$). When looking at the results, we can tell that the average attitude scores of both groups increased at approximately the same rate. Based on this, we can say that peer instruction has a positive effect on the attitudes of male and female group students towards mathematics, but there is no important difference between the two groups when looking at the effect of peer teaching. Furthermore, mathematics attitude corresponds directly with mathematics achievement as seen from the previous studies as well [23, p. 51], [293]. For instance, X. Ma and N. Kishor looked into 113 articles on the relationship between mathematics attitude and mathematics accomplishment. They concluded that a positive attitude towards mathematics directly relates to good results in mathematics. When we consider the current study, the positive attitudes towards mathematics might have caused a higher success rate while when we consider previous studies, a higher achievement rate can also positively affect mathematics attitudes hence gradually leading to better grades [292, p. 114].

In his research, A. F. Wong, D. J. Young, B. J. and Fraser concluded from his analysis that males and females had no notable differences in their attitudes in mathematics. This similarity in attitudes and mathematics achievement between males and females explained above comes about because of related childhood experiences, same employment requirements, and equality in classes when it comes to offering academic support to the students. From the academic achievement test results, we can also conclude that males and females have no important differences in their attitudes and achievement in mathematics. Furthermore, we can say that the outcome of the achievement test might affect the attitudes of the students on the mathematics scale as well. We can also note that at this level, both males and females have the same opportunities in classrooms with females gaining more confidence and

ability and can express themselves in the classrooms by actively engaging with the instructors in lessons. The learning process affects both male and female students in a similar way hence this might be the reason for having no significant differences between them [294].

Discussion of The PI student evaluation form

According to the survey, the participants stated that the courses in which peer teaching was applied attracted their attention and showed the highest average (4.725) in the questionnaire. Peer teaching to students was entertaining (4,623). S. M. Al-Hebaishi, in his study in 2015, in which the participants were 78 female graduate English teachers, the results of the questionnaire he made with the participants and the findings of this research were correlated. In the traditional teaching method, teachers are active and students are passive. The teacher lectures and asks questions, and the students are in a listener position and they try to answer the questions asked. In the lessons taught in this way, the teacher cannot affect the whole class and the lesson does not seem fun to the students in general. In peer education, which is an active learning method, students are active in the classroom, as they can actively tell their ideas that they have discussed with their peers and easily try to explain their correct or wrong answers to their peers. Learners stated that their contribution in class increased (4,449) [295]. Peer education increases students 'conceptualization success, decreases failure rates, increases student participation, and supports students' participation and attitudes in their courses (E. Mazur [5, p. 9]; Lucas [175, p. 222]; L. Porter, C. Bailey-Lee, & B. Simon [177]; W. Beekes [178], L. Deslauriers, E. Schelew, & C. Wieman [179]; B. Noonan, & C. R. Duncan [180]). Generally, as a result of the questionnaire, students reported that peer education created a positive atmosphere in the classroom, and this positive atmosphere increased learners' participation in the class and helped the students to have fun and understand the subjects better. They found that the peer teaching approach increased their motivation and helped to understand subjects. We can see these results in the previous studies (E. Mazur [5, p. 16]; C. H. Crouch & E. Mazur [7, p. 977]; T. Gok [17, p. 747]; L. Porter, C. Bailey-Lee, and B. Simon [177, p. 179]; R. N. Cortright, H. L. Collins, and S. E. DiCarlo [201, p. 111], M. J. Giuliodori, H. L. Lujan and S. E. DiCarlo [202, p.173]; L. Porter, C. B. Lee, B. Simon, & D. Zingaro [217, p. 51]; S. Ghosh & F. Renna [296]). Therefore, the students liked to apply the peer instruction method in mathematics lessons.

3. CONCLUSION

No important difference was found between the mathematics pretest success average scores of the treatment and control groups. This situation showed us that both groups are equal in terms of their prior knowledge on the subject. It is important that

the pre-implementation stages of the treatment and control group students are equal in order to better understand the effectiveness of peer education.

In many studies examined in the literature, it was observed that there was no important disparity between the pretest average scores of the treatment and control groups [15, p. 95], [19, p. 200].

There was a considerable meaningful difference obtained between the mathematics accomplishment average of the scores before the application (pretest) and the mathematics achievement scores after the application (posttest) of the learners in the treatment group where peer education was applied. Considering the mean rank and total of the difference scores, this disparity found is in favor of the positive ranks, that is, the posttest score. According to this result, peer education has a significant influence on increasing students' accomplishment in mathematics lesson trigonometry.

An important difference was found between the pre-application mathematics accomplishment mean scores (pretest) and the post-application mathematics achievement scores (posttest) of the control group students in which the traditional teaching method was applied. The difference is in favor of the final test score. According to these results, traditional teaching also has a positive influence on increasing students' mathematics accomplishment. In traditional teaching, the teacher is active and teaches the lesson with the method of direct instruction. The topics in the lesson are repeated by applying the question and answer method. Since the subject was learned, the posttest scores were higher. In peer education, active participation of students is in question. Group work makes the lesson fun and provides more permanent learning. For this reason, there was an enhancement in the success of the students in the control group where the traditional method was applied; however, this increase was not as high as the level of students in the treatment group where peer education was applied.

An important difference was found between the post-application mathematics achievement test scores (posttest scores) of the students in the treatment group where peer education was applied and the control group where traditional teaching was continued. The results are an answer to the first question of the study “What is the effect of peer instruction on academic achievement”. Treatment group learners’ posttest mathematics accomplishment average scores were higher than the average scores of the learners in the control group. This shows that peer education applied in the treatment group is a more effective method in increasing academic achievement. In this research study, the first hypothesis was "Peer instruction had a significant impact on the mathematics achievement of 9th-grade students." The results obtained in the study indicated that the hypothesis is correct. The most important benefit of peer education is that it benefits both students who are good and weak in the course. Students with good lessons learn the subject both while learning and teaching to their friends. In addition, a sense of responsibility develops. Weak students in the lesson, on the other hand, can ask their friends more easily the subjects they are hesitant to

ask their teachers. The results obtained have shown the same result as many studies in the literature (F. Demirel [15, p. 82]; G. Akay [23, p. 75]; R. E. Abdelkarim & E. Abuiyada [244]; A. B. Lacaba, J. D. Magalona & T. V. G. Lacaba [245]; E. A. Oloo, S. N. Mutsotso, & E. N. Masibo [246]; Y. Z. Olpak, S. Baltaci, & M. Arican [247]; S. Ouko, C. Aurah, & M. Amadalo [248]).

Abstract and difficult to understand topics can become interesting with peer education. For this reason, peer education is a method that can be used efficiently in mathematics lessons.

No important difference was observed between the attitudes towards mathematics of the learners in the treatment group in which peer teaching was applied and the control group, where traditional teaching was continued. This situation showed us that the attitudes of both groups towards mathematics before the application were equivalent to each other.

An important difference was found between the treatment group students' attitudes towards mathematics before and after the application, in which peer teaching was applied. While the attitude test mean score of the participants in the treatment group before the application was X = 3.08, the attitude test score average after the application was found to be X = 3.65. It can be said that peer instruction has an important influence on increasing students' attitudes in statistics in mathematics lessons.

A meaningful difference was found between the control group learners' attitudes towards mathematics before and after the application, where traditional teaching was applied. While the attitude test mean score of the learners in the control group before the application was X = 3.06, the attitude test score average after the application was found to be X = 3.13. In other words, the method applied changed the attitude. The results show that peer education increases the attitudes of 9th grade students towards mathematics lesson compared to traditional education. The results are the response to the second question of the study "What is the effect of peer instruction on students' attitudes towards mathematics lessons". In this research study, the second hypothesis was "Peer instruction had an important impact on the mathematics accomplishment of 9th-grade learners and their attitude towards mathematics." The results obtained in the study indicated that the hypothesis is correct. Previous studies in Literature support the outcomes of this study (J. B. Campit & R. M. Garin's [14, p. 15]; F. Demirel [15, p. 80]; G. Akay, 2011 [23, p. 77]; R. Abdelkarim, R. Abuiyada, & S. A. Siddiui [258, p. 1515]; O. C. Yavuz [259, p. 122]; K. M. P. Dias, C. M. Dias, & D. G. G. Sasaki [297]).

No important difference was found between math pretest average scores of male and female students in the experimental group in which peer education was applied. The pretest mean score of female students was X = 5.59, while the pretest mean score of males was X = 7.69. The average of the male students was higher. However, since

there was no important difference between the scores, the pretest mean scores of male and female learners were considered equal.

No meaningful difference was found between the mathematics posttest accomplishment average scores of male and female learners in the treatment group in which peer education was applied. While female students' posttest average score was X = 14.02, the posttest average score of males was X = 15.20. As a result of peer instruction, the success of male and female students increased, but this difference was not statistically significant. In other words, gender had no influence on the increase of academic accomplishment. The results obtained in this study show the same results as F. Demirel [15, p. 91] and G. Akay [23, p. 50].

No essential difference was found between pre-attitude scores of male and female students in the treatment group in which peer education was applied. While female students 'pre-attitude mean score was X = 3.20, male students' pre-attitude mean score was found to be X = 3.06. In other words, learners' attitudes towards the lesson before the implementation are close to each other.

No important difference was found between the final attitude scores of male and female learners in the treatment group in which peer education was applied. The final attitude score average of female students was X = 3.45, while the final attitude score mean of males was X = 3.30. According to the results, the attitudes of male and female students towards the lesson increased at the same rate, but this difference is not significant. In other words, gender has no effect on increasing attitude towards the lesson. Previous research also supports this result (G. Akay [23, p. 51]; B. Joseph Campit, Rodelio M. Garin [298]; L. A. Tartre & E. Fennema [299]).

4. SUGGESTIONS

In order for mathematics education, which is an important step of advancement in numerical courses, to be effective, it is necessary to ensure that students are free, critical, questioning, productive and creative. There is a need for learners to understand scientific knowledge and how this information is obtained, and to develop scientific process skills. There is an urgent necessity for the mathematics lesson to know the prior knowledge and to follow the subsequent conceptual changes when starting the lesson. Also, a positive attitude and curiosity is what a scientist should have. In mathematics lessons, it is of great importance to understand and tell the kind of attitude students have towards the lesson and to plan the teaching in a way that will provide a positive attitude. Positive attitude and curiosity towards the course also bring success. In making the course interesting, students' being active, interacting with their teachers and each other, and enriching the course with different materials is effective. All these important points should be taken into consideration while planning the mathematics lesson. Traditional methods are insufficient to provide this.

Recent studies reveal that active teaching is effective in acquiring these characteristics. However, it is known that active learning methods are not easy to apply to crowded classrooms. However, Peer Instruction is an active learning technique introduced by Eric Mazur and suitable for crowded classrooms [207], [261].

The research has emerged as one of the few studies in which the peer teaching method is used on trigonometry in mathematics teaching in Kazakhstan. It is important to apply the peer teaching method at different age and grade levels in mathematics teaching and compare it with the findings obtained from this study.

When using a different method such as a peer teaching method, students should be adequately informed about the activities to be done from the beginning so that the students feel safe during the application of the method and that they can have productive discussions when interacting with their peers.

Multiple choice concept questions can be prepared with compound answers (which one or which are correct question types). It has been observed that such questions lead students to think more deeply.

Since the peer teaching method requires dividing the units into subheadings, a careful preparation and planning should be done before the lesson, and attention should be paid to the preparation of lesson plans and concept questions, not just before the lessons, but in a way that covers the whole unit before the implementation.

It is thought that better results can be obtained with longer studies so as to be able to see a difference in students' attitudes towards the lesson. Therefore, the change in students' attitudes towards mathematics can be observed better with the applications of longer-term peer teaching methods.

During peer discussions, the teacher should go around with the students to check that the discussions are carried out in a way that suits their purpose.

In the implementation of the peer teaching method, using technology may be more effective in multiple-choice questions in which learners are told to respond first individually and then with their peers after course presentations. For this, it should not be forgotten that sufficient technological equipment should be provided in the teaching environment.

Peer teaching method should be introduced to prospective teachers at universities as an effective active learning approach and its various applications should be shown.

Introductory activities for the peer teaching method can be developed for teachers to obtain the required expertise and skills necessary for this method.

In the study, the effects of peer teaching methods in mathematics teaching in different dimensions compared to the traditional teaching method were investigated. When

looking from this perspective, it is useful to compare peer teaching methods with other methods used in mathematics teaching in the next period.

One of the important points in the peer instruction method is that learners work alongside each other and discuss conceptual questions in peer groups. From this viewpoint, these discussion conditions should be provided for the method to function fully. It will be useful to motivate students, especially at this stage, for the teacher performing the application to navigate between groups and direct discussions.

In order to support communication in peer groups, one week individual homework related to the same gains can be given, and group assignments the other week. At this stage, it will be possible to compare the results of homework done by individual and peer groups.

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APPENDIX

Appendix 1

Mathematics Achievement Test (MAT)

1. Change 150° to radian.

A) π B) $\frac{5\pi}{6}$ C) $\frac{7\pi}{4}$ D) $\frac{3\pi}{2}$ E) $\frac{2\pi}{3}$

2. Find the primary directed angle of $\frac{18\pi}{5}$

A. $\frac{\pi}{5}$ B. $\frac{3\pi}{5}$ C. $\frac{5\pi}{5}$ D. $\frac{7\pi}{5}$ E. $\frac{6\pi}{5}$

3. Sin 45° + Cos 45° = ?

A) ½ B) √3 C) √3/2 D) √2/2 E) √2

4. If tan x = ¾ then what is cos x ? x (ϵ 0,90)

A) 4/3 B) ¾ C) 1 D) 4/5 E) 3/5

5. $\frac{\frac{1}{Sinx}.Cotx}{\frac{1}{Sin^2x}} = ?$

A) 1 B) Sin x C) Cos x D) Tan x E) Cot x

6. $\frac{1-\tan^2 x}{1+\tan^2 x} = ?$

A) 1 B) 0 C) $\cos^2$x-Sin^2x D)1/Cosx E) $\frac{\cos x+\sin x}{2.\sin.\cos x}$

7. Which one of the followings is equal to $\cos(-25°)$?

A) $-\sin 115°$ B) $\sin 125°$ C) $-\cos 25°$ D) $\sin 115°$ E) $\sin 25°$

8. If $\sin\theta = -\frac{\sqrt{2}}{2}, \theta\epsilon(\pi, \frac{3\pi}{2})$ Find $\tan\theta$.

A)$\frac{\sqrt{2}}{2}$ B) -$\frac{\sqrt{2}}{2}$ C) 1 D) $\sqrt{2}$ E)- $\sqrt{2}$

9. Find the order of the signs of the given functions.

$$\sin 190, \cos 275, \tan 175, \cot 365$$

A- (-, +,-, +)

B- (-, +,-,-)

C- (+, +,-, +)

D- (-,-,-, +)

E- (-, +,+,+)

10. $m\text{A} = 28^0 35' 12''$ and $mB = 15^0 10' 40''$ equal $mA + mB = ?$

$a)43^0 45' 52''$ $b)44^0 44' 52''$ $c)43^0 44' 52''$

$d)44^0 45' 52''$ $e)43^0 44'''$

11. $\frac{1}{\sin 75°} - \frac{1}{\cos 75°} = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $-2\sqrt{2}$ D) $2\sqrt{6}$ E) $-2\sqrt{6}$

12. Let sinx.cosy= $\frac{1}{2}$ and siny.cosx= $\frac{1}{5}$ then evaluate $\frac{\sin(x+y)}{\sin(x-y)}$.

A) $\frac{7}{3}$ B) $\frac{3}{3}$ C) $\frac{2}{7}$ D) $\frac{3}{7}$ E) $\frac{5}{7}$

13. $\cos 75 \cdot \cos 15 + \sin 75 \cdot \sin 15 = ?$

$A)\frac{1}{2}$ $B)1$ $C)\frac{3+\sqrt{3}}{4}$ $D)\frac{\sqrt{3}}{2}$ $E)\frac{\sqrt{6}}{2}$

14. Sinx= $\frac{1}{2}$, Cosy= $\frac{2}{3}$. Tan(x+y)=?

A) $\frac{2+\sqrt{15}}{2\sqrt{3}-\sqrt{5}}$ B) $\frac{2-\sqrt{5}}{2\sqrt{3}}$ C)0 D)1 E)-1

15. cosx= $\frac{1}{\sqrt{3}}$ and x is in the fourth quadrant. What is sin2x?

$A)-\frac{2\sqrt{2}}{3}$ $B)-\frac{\sqrt{2}}{3}$ $C)\frac{\sqrt{6}}{3}$ $D)\frac{\sqrt{5}}{4}$ $E)\frac{2}{3}$

16. $\frac{2.\sin 25.\sin 65}{\cos 40} = ?$

A) 2 B) 4 C) 3 D) 5 E) 1

17. Which one of the following is true for 3. Quadrant ?

a) Sine (+) b) Cosine (+) c) Tangent (-) d) Cotangent (+) e) Secant(+)

18. $\frac{Cos23.Cos37 - Sin23.Sin37}{Sin17.\cos 77 - Sin77.Cos17} = ?$

A) $\frac{-1}{\sqrt{3}}$ B) $\frac{1}{2}$ C) $\frac{-1}{\sqrt{2}}$ D) $\frac{-\sqrt{3}}{2}$ E) $-\sqrt{3}$

19. Simplify the following expression.

$Sin(x+30)+Cos(x+60)$

A)Cosx B)Sinx C) Cos2x D) -2Cosx E)-Sinx

20. If $Cos2x = \frac{2}{5}$, then find $(Cos^4x - Sin^4x)$.

A) $\frac{3}{5}$ B) $\frac{1}{5}$ C) $\frac{5}{6}$ D) $\frac{5}{8}$ E) $\frac{2}{5}$

21. Sin (180-α)+Cos(90+α)- tan(360+α)+Cot(270-α)

A) 2Sinα-2tanα B) 2Sinα C) 1 D) 0 E) Cosα

22. Which one of the followings is the simplest form of

$$\frac{\sin(-\alpha)\cdot\cos(\pi+\alpha)}{2\cdot\sin(\pi-\alpha)}+\frac{\sin(\frac{3\pi}{2}-\alpha)\cdot\cos(\frac{\pi}{2}-\alpha)}{2\cdot\cos(\frac{\pi}{2}+\alpha)}?$$

A) sin (2α) B) cos(α) C) sin(α) D) tan(α) E) cos(α) – sin(α)

23. What is the maximum value of the sum, $3\sin^2\alpha+5\cos^2\alpha+2\cos\alpha$?

a) 10 b)6 c)7 d)8 e)9

24. What point corresponds to the angle $\frac{\pi}{2}$ on the unit circle?

A. (-1,0) B. (1,0) C. (-1,-1) D. (0,1) E.(0,0)

25. If $\frac{\pi}{4}<x<\frac{\pi}{2}$ then, which one of the following is true?

a)cosx < sinx < tanx b) sinx < cosx < tanx c) cosx < tanx < sinx

d) tanx < sinx < cosx e) sinx < tanx < cosx

Appendix 2

Attitudes towards Mathematics Survey (ATM)

	I Strongly Disagree	I Disagree	Neutral	I Agree	I Strongly Agree
1) I love mathematics lesson					
2) I am not comfortable in a mathematics lesson					
3) If there was no mathematics lesson, the world would be more enjoyable					
4) I enjoy discussing about mathematics with my friends					
5) I would love to have more hours of mathematics lessons					
6) I get bored whenever I study mathematics					
7) Mathematics makes me tired					
8) I like mathematics					
9) Time doesn't pass in a mathematics lesson					
10) I am scared of a mathematics exam					
11) Mathematics is exciting for me					
12) Mathematics is the scariest lesson					
13) I wouldn't be bored in a mathematics class even after many years					
14) I would study mathematics with more passion compared to other lessons					
15) Mathematics					

makes me uncomfortable					
16) Mathematics makes me scared					
17) Mathematics is a fun lesson					
18) I feel cheerful in a mathematics lesson					
19) In all the lessons, mathematics is the least liked					
20) I would like to spend more time doing mathematics					

Appendix 3

First mathematics achievement test (MAT)

1. Given that $\alpha = 28^o35'12"$ and $\theta = 15^o10'45"$ Find $2\alpha + \theta = ?$
   A. $72^o21'09"$ B. $72^o20'09"$ C. $72^o21'06"$ D. $73^o21'09"$
   E. $73^o22'09"$
2. How many degree is $\frac{5\pi}{6}$?
   A) 120° B) 150° C) 90° D) 560° E) 210°
3. Change 150° to radian.
   A) € B) $\frac{5\pi}{6}$ C) $\frac{7\pi}{4}$ D) $\frac{3\pi}{2}$ E) $\frac{2\pi}{3}$
4. Find the primary directed angle of 450^0
   a. 50^0
   b. 90^0
   c. 80^0
   d. 60^0
   e. 70^0
5. Find the primary directed angle of $\frac{18\pi}{5}$
   A. $\frac{\pi}{5}$ B. $\frac{3\pi}{5}$ C. $\frac{5\pi}{5}$ D. $\frac{7\pi}{5}$ E. $\frac{6\pi}{5}$
6. tan 0 – cot 90 + sin 60 . cos 60 = ?
   A) 1 / 2 B) √3 C) √2 D) $\frac{\sqrt{3}}{4}$ E) $\frac{\sqrt{2}}{2}$
7. Sin 45° + Cos 45° = ?
   A) ½ B) √3 C) √3/2 D) √2/2 E) √2
8. If tan x = ¾ then what is cos x ?
   A) 4/3 B) ¾ C) 1 D) 4/5 E) 3/5
9. Sin 0° + Cos 60° - Tan 0° = ?
   A) 1/2 B) –1 C) –1/2 D) 0 E) indefinite
10. $\frac{\frac{1}{Sinx}.Cotx}{\frac{1}{Sin^2x}} = ?$
    A) 1 B) Sin x C) Cos x D) Tan x E) Cot x

11. $\frac{1-\tan^2 x}{1+\tan^2 x} = ?$

A) 1 B) 0 C) $\cos^2 x - \sin^2 x$ D) 1/Cosx E) $\frac{\cos x + \sin x}{2.\sin.\cos x}$

12. Which one of the followings is equal to $\cos(-25°)$?

A) $-\sin 115°$ B) $\sin 125°$ C) $-\cos 25°$ D) $\sin 115°$ E) $\sin 25°$

13. a is an acute angle. If $\frac{3.sina+1}{4-5.sina} = \frac{2}{5}$, what is cosa ?

a. $\frac{3}{25}$
b. $2\sqrt{154}$
c. $\frac{9}{25}$
d. $2\sqrt{77}$
e. $3\sqrt{77}$

14. If $\sin\theta = -\frac{\sqrt{2}}{2}, \theta\epsilon(\frac{3\pi}{2}, 2\pi)$ Find $\tan\theta$.

A) $\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) 1 D) $\sqrt{2}$ E) $-\sqrt{2}$

15. Find the order of the signs of the given functions.

$\sin 190, \cos 275, \tan 175, \cot 365$

a. (-,+,-,+)
b. (-,+,-,-)
c. (+,+,-,+)
d. (-,-,-,+)
e. (-,+,+,+)

16. $m\text{A} = 28^0 35' 12''$ and $mB = 15^0 10' 40''$ equal $mA + mB = ?$

$a)43^0 45' 52''$ $b)44^0 44' 52''$ $c)43^0 44' 52''$
$d)44^0 45' 52''$ $e)43^0 44'''$

17. Which of the following is the greatest?

A) Tan 75 B) Cot75 C) Sec75
D) Sin75 E) Cos75

18. If cos(x-y)= $\frac{3}{5}$ and sinx= $\frac{4}{5}$, what is cosy?

A) 1 B) $\frac{1}{2}$ C) $\frac{7}{25}$ D) $\frac{23}{25}$ E) $\frac{-7}{25}$

19. $\frac{1}{\sin 75^\circ} - \frac{1}{\cos 75^\circ} = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $-2\sqrt{2}$ D) $2\sqrt{6}$ E) $-2\sqrt{6}$

20. Let sinx.cosy= $\frac{1}{2}$ and siny.cosx= $\frac{1}{5}$ then evaluate $\frac{\sin(x+y)}{\sin(x-y)}$.

A) $\frac{7}{3}$ B) $\frac{3}{3}$ C) $\frac{2}{7}$ D) $\frac{3}{7}$ E) $\frac{5}{7}$

21. $\cos 75 \cdot \cos 15 + \sin 75 \cdot \sin 15 = ?$

$A)\frac{1}{2}$ $B)1$ $C)\frac{3+\sqrt{3}}{4}$ $D)\frac{\sqrt{3}}{2}$ $E)\frac{\sqrt{6}}{2}$

22.. Sinx= $\frac{1}{2}$, Cosy= $\frac{2}{3}$. Tan(x+y)=?

A) $\frac{2+\sqrt{15}}{2\sqrt{3}-\sqrt{5}}$ B) $\frac{2-\sqrt{5}}{2\sqrt{3}}$ C)0 D)1 E)-1

23.cosx= $\frac{1}{\sqrt{3}}$ and x is in the fourth quadrant. What is sin2x?

$A)-\frac{2\sqrt{2}}{3}$ $B)-\frac{\sqrt{2}}{3}$ $C)\frac{\sqrt{6}}{3}$ $D)\frac{\sqrt{5}}{4}$ $E)\frac{2}{3}$

24. $\frac{2.\sin 25.\sin 65}{\cos 40} = ?$

A) 2 B) 4 C) 3 D) 5 E) 1

25. $\frac{Cos23.Cos37 - Sin23.Sin37}{Sin17.\cos 77 - Sin77.Cos17} = ?$

A) $\frac{-1}{\sqrt{3}}$ B) $\frac{1}{2}$ C) $\frac{-1}{\sqrt{2}}$ D) $\frac{-\sqrt{3}}{2}$ E) $-\sqrt{3}$

26. Simplify the following expression.

$Sin(x+30) + Cos(x+60)$

A)Cosx B)Sinx C) Cos2x D) -2Cosx E)-Sinx

27. If $Cos2x = \frac{2}{5}$, then find $(Cos^4 x - Sin^4 x)$.

A) $\frac{3}{5}$ B) $\frac{1}{5}$ C) $\frac{5}{6}$ D) $\frac{5}{8}$ E) $\frac{2}{5}$

28. Find the reference angle of 150°

A. 50° B. 250° C. 30° D. 130° E. 80°

29. Sin(180- 〈)+Cos(90+ 〈)- tg(360+ 〈)+Ctg(270- 〈)

A) 2Sin 〈-2tg 〈 B) 2Sin 〈 C) 1 D) 0 E) Cos 〈

30. . Which one of the followings is the simplest form of

$$\frac{\sin(-\alpha)\cdot\cos(\pi+\alpha)}{2\cdot\sin(\pi-\alpha)}+\frac{\sin(\frac{3\pi}{2}-\alpha)\cdot\cos(\frac{\pi}{2}-\alpha)}{2\cdot\cos(\frac{\pi}{2}+\alpha)}?$$

A) sin(2 〈) B) cos(〈) C) sin(〈) D) tan(〈) E) cos(〈) – sin(〈)

31. Evaluate the following $2\cos 140\cdot\sin(-40)$.

A) $-\sin 10$ B) $-\cos 10$ C) $\cos 10$ D) $\sin 40$ E) $\cos 20$

32. What is the maximum value of the sum, $3\sin^2\alpha+5\cos^2\alpha+2\cos\alpha$?

a)10 b)6 c)7 d)8 e)9

33. What is the ratio of sine in right triangle?

a) $\frac{opposite\ side}{adjacent\ side}$ b) $\frac{adjacent\ side}{hypotenuse}$ c) $\frac{adjacent\ side}{opposite\ side}$ d) $\frac{hypotenuse}{opposite\ side}$ e) $\frac{opposite\ side}{hypotenuse}$

34. Which one of the following is true for 3. Quadrant ?

a) Sine (+) b) Cosine (+) c) Tangent (-) d) Cotangent (+) e) Secant(+)

35. If $\frac{\pi}{4}<x<\frac{\pi}{2}$ then, which one of the following is true?

a) cosx < sinx < tanx b) sinx < cosx < tanx c) cosx < tanx < sinx

d) tanx < sinx < cosx e) sinx < tanx < cosx

36. Prove $(\cos 15^\circ+\sin 15^\circ)^2=\frac{3}{2}$

37. Prove $\cos^4 A-\sin^4 A=\cos 2A$

38. Show the $sin^2x+cos^2x=1$ by using the Pythagorean theorem

39. What point corresponds to the angle $\frac{\pi}{2}$ on the unit circle?

A. (-1,0) B. (1,0) C. (-1,-1) D. (0,1) E.(0,0)

Appendix 4

Distribution of items according to topics and bloom's taxonomy level before pilot study

1. UNITS OF ANGLE MEASURE UNDERSTNDING
2. UNITS OF ANGLE MEASURE UNDERSTNDING
3. UNITS OF ANGLE MEASURE UNDERSTNDING
4. PRIMARY DIRECTED ANGLES UNDERSTNDING
5. PRIMARY DIRECTED ANGLES UNDERSTNDING
6. TRIGONOMETRIC RATIOS UNDERSTANING
7. TRIGONOMETRIC RATIOS APPLICATION
8. TRIGONOMETRIC RATIOS ANALYSIS
9. TRIGONOMETRIC RATIOS UNDERSTANDING
10. TRIGONOMETRIC IDENTITIES APPLICATION
11. TRIGONOMETRIC IDENTITIES APPLİCATİON
12. TRIGONOMETRIC FUNCTIONS UNDERSTANDING
13. CALCULATING TRIGONOMETRIC VALUES ANALYSIS
14. CALCULATING TRIGONOMETRIC VALUES APPLICATION
15. TRIGONOMETRIC FUNCTIONS ANALYSIS
16. UNİTS OF ANGLE MEASURE UNDERSTANDİNG
17. TRIGONOMETRIC FUNCTIONS ANALYSIS
18. TRIGONOMETRIC FORMULAS ANALYSIS
19. TRIGONOMETRIC FORMULAS APPLICATION
20. TRIGONOMETRIC FORMULAS APPLICATION
21. TRIGONOMETRIC FORMULAS UNDERSTANDİNG
22. TRIGONOMETRIC FORMULAS ANALYSIS
23. TRIGONOMETRIC FORMULAS ANALYSIS
24. TRIGONOMETRIC FORMULAS APPLICATION
25. TRIGONOMETRIC FORMULAS APPLICATION
26. TRIGONOMETRIC FORMULAS APPLICATION
27. TRIGONOMETRIC FORMULAS APPLICATION
28. TRIGONOMETRIC FORMULAS UNDERSTANDING
29. TRIGONOMETRIC FORMULAS APPLICATION
30. TRIGONOMETRIC FORMULAS ANALYSIS
31. TRIGONOMETRIC FORMULAS APPLICATION
32. TRIGONOMETRIC FUNCTIONS ANALYSIS
33. TRIGONOMETRIC RATIOS REMEMBERING
34. TRIGONOMETRIC FUNCTIONS REMEMBERING
35. TRIGONOMETRIC FUNCTIONS ANALYSIS
36. TRIGONOMETRIC FORMULAS CREATING
37. TRIGONOMETRIC FORMULAS CREATING
38. TRIGONOMETRIC IDENTITIES EVALUATING
39. UNITS OF ANGLE MEASURE REMEMBERING

Appendix 5

Mail sent to experts to check the questions

Dear expert.

I am Serkan Kaymak is a PhD student in mathematics education at SDU University in Kazakhstan. I would like to create a trigonometry achievement test to apply in the 9th grades related to the thesis position. Attached in the attachment;

1. Whether the questions are appropriate for the level of Grade 9 students

2. Whether the questions are classified correctly according to Bloom Taxonomy

3. General comments about the questions

4. Let you know what needs to be removed from the questions

5. Notify you of changes to the questions

I ask you.

Please also let us know the questions you want added.

Yours truly

Serkan Kaymak